अनुश्रुत
1. आपने कितने वर्षों तक शुक्ल पूर्ण है? इस पाठ्य पुस्तिका से पढ़ते हैं कि शुक्ल पूर्ण 35 साल तक होता है और 65 साल तक रहने के लिए भी किसी समय तक नहीं है। सबसे कम साल 35 साल तक 65 साल तक आपकी उम्र होनी चाहिए।

2. इस लेख के अन्तर्गत प्रश्न अत्यधिक है। इसलिए, उत्तर देने के लिए आपको अधिक समय लगना पड़ेगा।

3. इस पाठ्य पुस्तिका के साथ अनेक विषयों का प्रश्न पूछने के लिए इस पाठ्य सामग्री का उपयोग किया जाता है।

4. अन्य अवयवों के साथ इस पाठ्य पुस्तिका के साथ अनेक विषयों का प्रश्न पूछने के लिए इस पाठ्य सामग्री का उपयोग किया जाता है।

5. इस पाठ्य पुस्तिका के साथ अनेक विषयों का प्रश्न पूछने के लिए इस पाठ्य सामग्री का उपयोग किया जाता है।

6. इस पाठ्य पुस्तिका के साथ अनेक विषयों का प्रश्न पूछने के लिए इस पाठ्य सामग्री का उपयोग किया जाता है।
INSTRUCTIONS

1. You have opted for English as medium of Question Paper. This Test Booklet contains seventy five (75) Part A-25 Part B-30 Part C-20 Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 20 questions from part A, B and C respectively. If more than required number of questions are answered, only first 15, 20, 20 questions in parts A, B and C respectively, will be taken up for evaluation.

2. OMR answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that there are not torn or mutilated. If it is so, you may request the invigilator to change the booklet of the same code. Likewise, check the OMR answer sheet also. Sheets for rough work have been appended to the test booklet.

3. Write your Roll No., Name and Serial Number of this Test Booklet on the OMR Answer sheet in the space provided. Also put your signatures in the space provided.

4. You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the Answer Sheet, failing which the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.

5. Each question in Part A carries 2 marks, Part B 3.5 marks, Part C 5 marks respectively. There will be negative markings @ 25% (Part A) 0.50 marks, Part B 0.875 marks and Part C 1.25 marks for each wrong answer.

6. Below each question in Part A, B and C four alternatives or responses are given. Only one of those alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer.

7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.

8. Candidate should not write anything anywhere except on answer sheets or copy of paper.

9. Use of calculator is not permitted.

10. After the test is over, at the perforation point, tear the OMR answer sheet and hand over the original OMR answer sheet to the invigilator and retain the carbonless copy for your record.

11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.
### Table of Fundamental Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_e$</td>
<td>$9.11\times 10^{-31}$</td>
<td>kg</td>
</tr>
<tr>
<td>$h$</td>
<td>$6.63\times 10^{-34}$</td>
<td>Js</td>
</tr>
<tr>
<td>$e$</td>
<td>$1.6\times 10^{-19}$</td>
<td>C</td>
</tr>
<tr>
<td>$k_B$</td>
<td>$1.38\times 10^{-23}$</td>
<td>J/K</td>
</tr>
<tr>
<td>$c$</td>
<td>$3.0\times 10^8$</td>
<td>m/s</td>
</tr>
<tr>
<td>$G$</td>
<td>$6.67\times 10^{-11}$</td>
<td>N m$^2$/kg$^2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$1.097\times 10^{-7}$</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$N_A$</td>
<td>$6.023\times 10^{23}$</td>
<td>mole$^{-1}$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>$8.854\times 10^{-12}$</td>
<td>F m$^{-1}$</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>$4\pi\times 10^{-7}$</td>
<td>H m$^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>$8.314$</td>
<td>J K$^{-1}$ mole$^{-1}$</td>
</tr>
</tbody>
</table>

**Planck's constant:** $h$

**Charge of electron:** $e$

**Bohr magneton:** $\mu_B$

**Speed of light:** $c$

**Newton's constant:** $G$

**Rydberg constant:** $\alpha$

**Avogadro's number:** $N_A$

**Permittivity of free space:** $\varepsilon_0$

**Permeability of free space:** $\mu_0$

**Boltzmann constant:** $k_B$
BHAG/PART-A

1. **The diameters of the pinholes of two otherwise identical cameras A and B are 500 µm and 260 µm, respectively. Then the image in camera A will be:**
   1. sharper than in B
   2. darker than in B
   3. less sharp and brighter than in B
   4. sharper and brighter than in B

2. **If \( y = 2x^2 \) and \( y = 4x \) are two curves, then how many points of intersection are there?**
   1. exactly one point
   2. exactly two points
   3. more than two points
   4. no point at all

3. **If \( x^2 + y^2 + 4x^2 + 5y^2 + 17y^2 - 18x^2 + 19y^2 \) then what is the answer?**
   1. -5
   2. 12
   3. 95
   4. 190

4. **Find the area of the composite region in the figure below:**

5. **At a birthday party, every child gets 2 chocolates, every mother gets 1 chocolate, while no father gets a chocolate. In total 69 persons get 70 chocolates. If the number of children is half of the number of mothers and fathers put together, then how many fathers are there?**
   1. 22
   2. 23
   3. 24
   4. 69

6. **A rectangular photo frame of size 30 cm x 40 cm has a photograph mounted at the centre leaving a 5 cm border all around. The area of the border is**
   1. 1000 cm²
   2. 350 cm²
   3. 450 cm²
   4. 700 cm²

7. **A rectangular 30 cm x 40 cm photo of 20 cm x 10 cm is to be cut from the paper. What is the maximum area that can be obtained?**

8. **If \( T = 20 \text{ km} \) and \( T = 10 \text{ km} \) then what is the answer?**
   1. 20 km
   2. 30 km
   3. 10 km
   4. 20 km

9. **What is the value of \( \sqrt{2} - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2 \)?**
   1. -5
   2. 12
   3. 95
   4. 190

10. **If \( B = H \) then what is the answer?**

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---
6. A tourist drives 20 km towards east, turns right and drives 4 km, then drives 6 km towards west. He then turns to his left and drives 4 km and finally turns right and drives 14 km. Where is he from his starting point?

1. 6 ft
2. 10 ft
3. 12 ft
4. 36 ft

8. A circular running track has six lanes, each 1 m wide. How far ahead (in metres) should the runner in the outermost lane start from, so as to cover the same distance in one lap as the runner in the innermost lane?

1. 6 ft
2. 10 ft
3. 12 ft
4. 36 ft

9. (a) In a five-term progression, the last term is 100. If the first term is 3, find the common difference.
(b) In a five-term progression, the first term is 100. If the last term is 3, find the common difference.

1. 10
2. 15
3. 20
4. 25

9. For the following set of observed values 60, 65, 65, 70, 70, 70, 70, 72, 82, 85, 90, 95, 95, 100, 100, 100, 100.

1. mode < median < mean
2. mode < mean < median
3. mean < median < mode
4. median < mode < mean

10. If \( D = ABC + BCA + CAB \) and \( A, B \) and \( C \) are decimal digits, then \( D \) is divisible by

1. 37 and 29
2. 37 but not 29
3. 29 but not 37
4. neither 29 nor 37
11. Two solutions X and Y containing ingredients A, B and C in proportions x:y:z, and c:d:e, respectively, are mixed. For the resultant mixture to have A, B and C in equal proportion, it is necessary that

\[ \frac{x}{y} = \frac{d}{e} \quad \text{and} \quad \frac{y}{z} = \frac{c}{d} \]

12. Scatter plots for pairs of observations on the variables \( x \) and \( y \) in samples A and B are shown in the figure.

Which of the following is suggested by the plots?
1. Correlation between \( x \) and \( y \) is stronger in A than in B.
2. Correlation between \( x \) and \( y \) is absent in B.
3. Correlation between \( x \) and \( y \) is weaker in A than in B.
4. \( y \) and \( x \) have a cause–effect relationship in A, but not in B.

13. A tank is filled with water. Which of the following processes is the slowest?
1. Evaporation
2. Condensation
3. Boiling
4. Freezing

14. An ideal pendulum oscillates with angular amplitude of 30° from the vertical. If it is observed at a random instant of time, its angular deviation from the vertical is most likely to be
1. 0°
2. ±10°
3. ±20°
4. ±30°

15. Which of the following is an example of a 'NODding'?
1. SALAD
2. SOUPS
3. RASAM
4. ONION
18. The lift (upward force due to air) generated by the wings and engines of an aircraft is
1. positive (upwards) while landing and negative (downwards) while taking off,
2. negative (downwards) while landing and positive (upwards) while taking off,
3. negative (downwards) while landing as well as while taking off,
4. positive (upwards) while landing as well as while taking off.

19. Areas of three parts of a rectangle are given in unit of cm². What is the total area of the rectangle?

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
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</table>

1. 18  2. 24  3. 36  4. 108

19. Find the missing figure in the following sequence.

<table>
<thead>
<tr>
<th></th>
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<th>?</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

1. 2. 3. 4. 5. 6. 7. 8. 9.
20. In triangle ABC, AB = 11, BC = 61, AC = 60, and O is the mid-point of BC. Then AO is

\[
\text{[Not to scale]}
\]

1. 12.5
2. 24.0
3. 30.5
4. 36.0

21. Three identical spin-\(1/2\) particles of mass \(m\) are confined to a one-dimensional box of length \(l\), but are otherwise free. Assuming that they are non-interacting, the energies of the lowest two energy eigenstates, in units of \(\hbar^2/2mL^2\), are

1. 3 and 6
2. 6 and 9
3. 6 and 11
4. 3 and 9

22. भाग/B-III

23. विशेषतः एक-ही, परिपक्व: \(1/2\) वाले तीन कण लंबाई \(l\) के एक-सीमी अंतरांक में सरकारित, पर अन्य स्थल है। अन्य स्थल है कि \(\Psi \) अपनी विशेषता वितरण है, \(\hbar^2/2mL^2\) के लंबाई तंत्र ओर-आयामपूर्ण अनुसारों की जगह है।

1. 3 तथा 6
2. 6 तथा 9
3. 6 तथा 11
4. 3 तथा 9

24. इतिहासी अनुमान को अनुप्रयोग करते समय आयाम की अनप्रयोग जड़ में लंबाई अनुमान जड़ में है। वीट अनुमान की लंबाई अनुमान जड़ में है, कि वीट है कर्मिक उदाहरण अनुमान अनुमान करके अनुमान, इसी अनुसार में है?

1. \(e^{-\frac{1}{2m}l}\)
2. \(e^{\frac{1}{2m}l}\)
3. \(\tanh\left(\frac{\hbar}{2m}\right)\)
4. \(\frac{1}{2}\cosech\left(\frac{\hbar}{2m}\right)\)
24. The vibrational motion of a diatomic molecule may be considered to be that of a simple harmonic oscillator with angular frequency \( \omega \). If a gas of these molecules is at a temperature \( T \), what is the probability that a randomly picked molecule will be found in its lowest vibrational state?

\[
1. \quad 1 - e^{-\frac{\hbar \omega}{kT}} \\
2. \quad \sqrt{\frac{\hbar \omega}{2kT}} \\
3. \quad \text{tanh} \left( \frac{\hbar \omega}{2kT} \right) \\
4. \quad \frac{1}{2} \text{coth} \left( \frac{\hbar \omega}{2kT} \right)
\]

25. Consider an ideal Fermi gas in a grand canonical ensemble at a constant chemical potential. The variance of the occupation number of the single particle energy level with mean occupation number \( \bar{n} \) is

\[
1. \quad \chi(1 - \bar{n}) \\
2. \quad \sqrt{\bar{n}} \\
3. \quad \bar{n} \\
4. \quad \frac{1}{\sqrt{\bar{n}}}
\]

26. Consider the following circuit, consisting of an RS flip-flop and two AND gates.

Which of the following connections will allow the entire circuit to act as a JK flip-flop?

1. connect \( Q \) to pin 1 and \( \overline{Q} \) to pin 2
2. connect \( Q \) to pin 2 and \( \overline{Q} \) to pin 1
3. connect \( Q \) to K input and \( \overline{Q} \) to J input
4. connect \( Q \) to J input and \( \overline{Q} \) to K input

27. Consider the following connections on \( Y(A, B, C) \) for the same circuit given above, where \( A, B \) and \( C \) are binary variables.

\[
\begin{array}{ccc|c}
A & B & C & Y \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
\end{array}
\]

In which \( Y \) is this circuit of the same state as the last?

1. \( Y = ABC + \overline{ABC} + A\overline{BC} + AB\overline{C} \)
2. \( Y = ABC + \overline{ABC} + A\overline{BC} + AB\overline{C} \)
3. \( Y = ABC + A\overline{BC} + \overline{ABC} + AB\overline{C} \)
4. \( Y = A\overline{BC} + ABC + A\overline{BC} + \overline{ABC} \)

27. The truth table below gives the value \( Y(A, B, C) \) where \( A, B \) and \( C \) are binary variables.

\[
\begin{array}{ccc|c}
A & B & C & Y \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
\end{array}
\]
The output \( Y \) can be represented by:
1. \( Y = \overline{ABC} + \overline{AB}C + \overline{ABC} + ABC \)
2. \( Y = \overline{ABC} + \overline{AB}C + \overline{ABC} + ABC \)
3. \( Y = \overline{ABC} + \overline{AB}C + \overline{ABC} + ABC \)
4. \( Y = \overline{ABC} + \overline{AB}C + \overline{ABC} + ABC \)

28. Which of the following graphs best describes the output waveform?

Which of the following graphs best describes the output waveform?

29. In the circuit, a sinusoidal signal is an input to the following circuit.

A sinusoidal signal is an input to the following circuit.

\[
\text{input} \quad V_i \quad \text{output} \quad V_o
\]

\[
\begin{array}{c}
\text{input} \\
V_i \\
\end{array}
\begin{array}{c}
\text{output} \\
V_o \\
\end{array}
\]
29. A sinusoidal voltage having a peak value of $V_0$ is an input to the following circuit, in which the DC voltage is $V_0$.

Assuming an ideal diode, which of the following best describes the output waveform?

30. Assume $e^A$ is the exponential matrix of $e^A$. The value of $A$ is $\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix}$. The product of the other two eigenvalues of $e^A$ is

1. $e^{2a}$
2. $e^{-2a}$
3. $e^a$
4. 1

31. Let $f(x) = 1 + 5x + 3x^2$ be a polynomial. The polynomials $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = \frac{1}{2}(3x^2 - x)$ can be expressed as $f(x) = \sum c_i p_i(x)$ as $1$. The answer is

1. $1/4$
2. $1/2$
3. 2
4. 4
31. The polynomial \( f(x) = 1 + 5x + 3x^2 \) is written as a linear combination of the Legendre polynomials \( \left\{ P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{3}{2}(3x^2 - 1) \right\} \) as \( f(x) = \sum c_n P_n(x) \). The value of \( c_0 \) is
   1. \( 1/4 \)  
   2. \( 1/2 \)  
   3. 2  
   4. 4.

32. \( c \) is a constant for the line \( \frac{x}{a} + \frac{y}{b} = 1 \) is always true.
   1. \( 4 \)  
   2. \( 4i \)  
   3. \( 2i \)  
   4. 0.

33. The value of the integral \( \int_C \frac{dx}{y + i} \) where \( C \) is a circle of radius \( R \) traversed counter-clockwise, with centre at \( z = 0 \), is
   1. \( R \)  
   2. \( 2i \)  
   3. \( 2R \)  
   4. 0.

34. The integral \( I = \int_C e^{z^2}dz \) is evaluated from the point \( (-1,0) \) to \( (1,0) \) along the contour
   \( C \), which is an arc of the parabola \( y = x^2 - 1 \), as shown in the figure.

35. A particle of mass \( m \), moving along the \( x \)-direction, experiences a damping force \(-\gamma v^2\), where \( y \) is a constant and \( v \) is its instantaneous speed. If the speed at \( t = 0 \) is \( v_0 \), the speed at time \( t \) is
   1. \( v_0 e^{-\frac{\gamma t^2}{m}} \)  
   2. \( \frac{v_0}{1 + \ln\left(\frac{2\gamma t}{m}\right)} \)  
   3. \( \frac{m}{\gamma + m + \frac{1}{2}} \)  
   4. \( \frac{v_0}{1 + \frac{2\gamma t}{m}} \).

36. In terms of arbitrary constants \( A \) and \( B \), the general solution to the differential equation \( x^2 \frac{dy}{dx} + 5x \frac{dy}{dx} + 3y = 0 \) is
   1. \( y = A + Bx^3 \)  
   2. \( y = Ax^3 + Bx^3 \)  
   3. \( y = Ax + Bx^3 \)  
   4. \( y = Ax^3 + Bx^3 \).
36. In the attractive Kepler problem described by the central potential \( V(r) = -k/r \) (where \( k \) is a positive constant), a particle of mass \( m \) with a non-zero angular momentum \( \ell \) can never reach the centre due to the centrifugal barrier. If we modify the potential to
\[
V'(r) = \frac{k}{r} - \frac{\beta}{r^2}
\]
we find that there is a critical value of the angular momentum \( \ell_c \) below which there is no centrifugal barrier. This value of \( \ell_c \) is
1. \([12k m^2 \beta]^{1/2}\)
2. \([12k m^2 \beta]^{-1/2}\)
3. \([12k m^2 \beta]^{1/4}\)
4. \([12k m^2 \beta]^{-1/4}\)

37. The time period of a particle of mass \( m \), undergoing small oscillations around \( x = 0 \), in the potential \( V = V_0 \cosh (\frac{x}{a}) \), is
\[
T = \pi \sqrt{\frac{2mV_0}{m_n^2}}
\]
1. \( \pi \sqrt{\frac{m}{V_0}} \)
2. \( 2\pi \sqrt{\frac{m}{m_n^2}} \)
3. \( 2\pi \sqrt{\frac{m}{V_0}} \)
4. \( 2\pi \sqrt{\frac{m_n}{m}} \)

38. The potential \( V(x) = -k/r \) (where \( k \) is a constant) is non-separable in the coordinate \( r \). The potential \( V = V_0 \cosh (\frac{x}{a}) \), is
\[
V = \frac{k}{r} - \frac{\beta}{r^2}
\]
1. \([12k m^2 \beta]^{1/2}\)
2. \([12k m^2 \beta]^{-1/2}\)
3. \([12k m^2 \beta]^{1/4}\)
4. \([12k m^2 \beta]^{-1/4}\)

39. Consider the decay \( A \to B + C \) of a relativistic spin- \( \frac{1}{2} \) particle \( A \). Which of the following statements is true in the rest frame of the particle \( A \)?
1. The spin of both \( B \) and \( C \) must be \( \frac{1}{2} \).
2. The sum of the masses of \( B \) and \( C \) is greater than the mass of \( A \).
3. The energy of \( B \) is uniquely determined by the masses of the particles.
4. The spin of both \( B \) and \( C \) may be integral.
39. Two current-carrying circular loops, each of radius \( R \), are placed perpendicular to each other as shown in the figure below.

The loop in the xy-plane carries a current \( I_0 \) while that in the yz-plane carries a current \( 2I_0 \). The resulting magnetic field \( \mathbf{B} \) at the origin is

1. \( \frac{I_0}{2\pi} (2\mathbf{f} + \mathbf{e}) \)
2. \( \frac{5I_0}{2\pi} (2\mathbf{f} - \mathbf{e}) \)
3. \( \frac{5I_0}{2\pi} (-2\mathbf{f} + \mathbf{e}) \)
4. \( \frac{5I_0}{2\pi} (-2\mathbf{f} - \mathbf{e}) \)

40. \( \text{Displacement } \mathbf{B} = q \mathbf{E} \) of the two circular loops is given by \( \mathbf{B} = \mathbf{E} \times \mathbf{r} \), where \( \mathbf{E} \) is the electric field and \( \mathbf{r} \) is the position vector from the source to the observation point. The direction of \( \mathbf{B} \) is perpendicular to both \( \mathbf{E} \) and \( \mathbf{r} \).

41. \( \text{Young's (Young)} \) is to a double-slit experiment, where light of wavelength \( \lambda \) is incident on two closely spaced slits. The interference pattern on the screen is observed.

42. An electric dipole of dipole moment \( \mathbf{p} = q \mathbf{d} \) is placed at the origin in the vicinity of two charges \( +q \) and \( -q \) at \((L, 0)\) and \((-L, 0)\), respectively, as shown in the figure below.

The electrostatic potential at the point \((L/2, 0)\) is

\[
\phi = \frac{2q}{\pi \epsilon_0 (L^2 + x^2 + y^2)}
\]

43. A monochromatic and linearly polarised light is used in a Young's double slit experiment. A linear polariser, whose pass axis is at an angle 45° to the polarisation of the incident wave, is placed in front of one of the slits. If \( I_{max} \) and \( I_{min} \), respectively, denote the maximum and minimum intensities of the interference pattern on the screen, the visibility, defined as the ratio \( I_{max}/I_{min} \), is

\[
\frac{I_{max}}{I_{min}} = \frac{2}{\sqrt{3}}
\]
42. An electromagnetic wave propagates in a nonmagnetic medium with relative permittivity $\varepsilon = 4$. The magnetic field for this wave is

$$B(x, y) = \frac{1}{\varepsilon} H_0 \cos(\omega t - ax - ay\sqrt{3})$$

where $H_0$ is a constant. The corresponding electric field $E(x, y)$ is

$$E(x, y) = \frac{1}{\varepsilon} H_0 \cos(\omega t - ax - ay\sqrt{3})$$

1. $\frac{1}{\varepsilon} \mu_0 H_0 (\sqrt{3} i + j) \cos(\omega t - ax - ay\sqrt{3})$
2. $\frac{1}{\varepsilon} \mu_0 H_0 (\sqrt{3} i + j) \cos(\omega t - ax - ay\sqrt{3})$
3. $\frac{1}{\varepsilon} \mu_0 H_0 (\sqrt{3} i - j) \cos(\omega t - ax - ay\sqrt{3})$
4. $\frac{1}{\varepsilon} \mu_0 H_0 (\sqrt{3} i - j) \cos(\omega t - ax - ay\sqrt{3})$

43. The ground state energy of an anisotropic harmonic oscillator described by the potential $V(x, y, z) = \frac{1}{2} m\omega_x^2 x^2 + 2 m\omega_y^2 y^2 + 8 m\omega_z^2 z^2$ (in units of $\hbar\omega$) is

1. $\frac{1}{2} \hbar \omega$ 2. $\frac{3}{2} \hbar \omega$
3. $\frac{5}{2} \hbar \omega$ 4. $\frac{1}{2} \hbar \omega$

44. If the potential $V(x, y) = \frac{1}{2} m\omega_x^2 x^2 + 2 m\omega_y^2 y^2 + 8 m\omega_z^2 z^2$ is applied to the system, the energy levels are

1. $\frac{1}{2} \hbar \omega$ 2. $\frac{3}{2} \hbar \omega$
3. $\frac{5}{2} \hbar \omega$ 4. $\frac{1}{2} \hbar \omega$

45. The product $\Delta x \Delta p$ of uncertainties in the position and momentum of a simple harmonic oscillator of mass $m$ and angular frequency $\omega$ is $\frac{\hbar}{2}$. The value of the product $\Delta x \Delta p$ in the state $e^{-i\frac{1}{2}x^2}$ (where $x$ is a constant and $p$ is the momentum operator) is

1. $\frac{\hbar}{2} \sqrt{\frac{\omega}{\pi m}}$ 2. $\hbar$
3. $\frac{\hbar}{2}$ 4. $\frac{2\hbar}{\omega m}$

46. The wavefunction of the electron in a hydrogen atom is

$$\psi(r) = \frac{1}{\sqrt{\pi r}} \phi_{1s}(r) + \frac{\sqrt{2}}{\sqrt{3}} \phi_{2s}(r) + \frac{\sqrt{2}}{\sqrt{3}} \phi_{3s}(r)$$

where $\phi_{1s}(r)$ are the eigenstates of the Hamiltonian in the standard notation. The expectation value of the energy in this state is

1. $-10.8 \text{ eV}$ 2. $-6.2 \text{ eV}$
3. $-9.5 \text{ eV}$ 4. $-5.1 \text{ eV}$
PART/C

46. अंकह दर्शण (100, 10.0, 0.0, 9.9, 9.8, 9.8, 9.9, 9.9, 9.9)
के मानक विचलन दिखाए नाकस है?
1. 0.10
2. 0.07
3. 0.01
4. 0.04

47. The standard deviation of the following set of data
(100, 10.0, 0.0, 9.9, 9.8, 9.8, 9.9, 9.9, 9.9, 9.9)
is nearest to
1. 0.10
2. 0.07
3. 0.01
4. 0.04

48. The excited state (n = 4, l = 2) of an electron in an atom may decay to one or more of the lower energy levels shown in the diagram below.

n = 4
l = 2

n = 3
l = 0
l = 1
l = 2

n = 2
l = 1

Of the total emitted light, a fraction 1/4 comes from the decay to the state (n = 2, l = 1). Based on selection rules, the fractional intensity of the emission line due to the decay to the state (n = 3, l = 1), will be
1. 3/4
2. 1/2
3. 1/4
4. 0

49. इक्सेनियम को अवधक करेगा 1 cm² है। 0.1 nm की लंबाई में (विद्युत लांच l = 500 nm पर) उसके लिए निर्देश वोल्ट के गद्द (mode) संख्या होगी
1. $10^3$
2. $10^7$
3. $10^{10}$
4. $10^{14}$

50. अरेराइन रेजीन के (BaTiO₃) केसेल (क्रिस्टल) एक गार्ज़नक पेटेंटरियट संरचना है, जहाँ Ba⁺⁺ आकार एक झड़ी दिन के शीर्ष के लिए है, O²⁻ आकार तकनीक के फैक्टर में अच्छी तरह से है। क्रिस्टल के प्रकार के वैक्लाई वोल्ट गद्द (mode) की संख्या होगी
1. 3-B-H
50. Barium Titanate (BaTiO₃) crystal has a cubic perovskite structure, where the Ba²⁺ ions are at the vertices of a unit cube, the O²⁻ ions are at the centres of the faces while the Ti⁴⁺ is at the centre. The number of optical phonon modes of the crystal is

51. The dispersion relation of optical phonons in a cubic crystal is given by \( \omega(k) = \omega_0 - \alpha k^2 \) where \( \omega_0 \) and \( \alpha \) are positive constants. The contribution to the density of states due to these phonons with frequencies just below \( \omega_0 \) is proportional to

52. A silicon crystal is doped with phosphorus atoms. (The binding energy of a H atom is 13.6 eV, the dielectric constant of silicon is 12 and the effective mass of electrons in the crystal is 0.4 mₑ.) The gap between the donor energy level and the bottom of the conduction band is nearest to

53. Assume that pion-nucleon scattering at low energies, in which isospin is conserved, is described by the effective interaction potential \( V(r) = F(r) l_x l_y \), where \( F(r) \) is a function of the radial separation \( r \) and \( l_x \) and \( l_y \) denote, respectively, the isospin vectors of a pion and the nucleon. The ratio \( \sigma_{1/2} / \sigma_{3/2} \) of the scattering cross-sections corresponding to total isospins \( I = 3/2 \) and \( 1/2 \), is

54. The Brillouin function (Fermi function) of the conduction band is given by \( \frac{1}{1 + e^{(E-E_F)/k_B T}} \).
54. A nucleus decays by the emission of a gamma ray from an excited state of spin-parity $2^+$ to the ground state with spin-parity $0^+$. What is the type of the corresponding radiation?
1. magnetic dipole
2. electric quadrupole
3. electric dipole
4. magnetic quadrupole

55. The low lying energy levels due to the vibrational excitations of an even-even nucleus are shown in the figure below.
57. A $4 \times 4$ complex matrix $A$ satisfies the relation $A^T A = I$, where $I$ is the $4 \times 4$ identity matrix. The number of independent real parameters of $A$ is

1. 32  
2. 16  
3. 12  
4. 64

58. The contour $C$ for the contour integral $\int_C \frac{dz}{z^2 - 1}$ in the complex $z$-plane is shown in the figure below.

This integral is equivalent to an integral along the contours

1.  
2.  
3.  
4. 

59. The contour $C$ for the contour integral $\int_C \frac{dz}{z^2 - 1}$ in the complex $z$-plane is shown in the figure below.

59. The value of the integral $\int_0^1 x^2 dx$, evaluated using the trapezoidal rule with a step size of 0.2, is

1. 0.30  
2. 0.39  
3. 0.34  
4. 0.27

60. The contour $C$ for the contour integral $\int_C \frac{dz}{z^2 - 1}$ in the complex $z$-plane is shown in the figure below.

60. The motion of a particle in one dimension is described by the Lagrangian $L = \frac{1}{2} (\dot{x}^2 - \dot{x}^2 x^2)$ in suitable units. The value of the action along the classical path from $x = 0$ at $t = 0$ to $x = x_0$ at $t = T_0$ is

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5-B-H
61. The Hamiltonian of a classical onedimensional harmonic oscillator is
\[ H = \frac{1}{2}(p^2 + x^2), \] in suitable units. The total time derivative of the dynamical variable \((p + \sqrt{2}x)\) is

1. \(\sqrt{2}p - x\)
2. \(p - \sqrt{2}x\)
3. \(p + \sqrt{2}x\)
4. \(x + \sqrt{2}p\)

62. In an inertial frame, uniform electric and magnetic fields \(E\) and \(B\) are perpendicular to each other and satisfy \(|E|^2 - |B|^2 = 29\) (in suitable units). In another inertial frame, which moves at a constant velocity with respect to the first frame, the magnetic field is \(2\sqrt{2} E\). In the second frame, an electric field consistent with the previous observations is

1. \(\sqrt{2}(1 + f)\)
2. \(7(1 + f)\)
3. \(\frac{7}{3}(1 + f)\)
4. \(7(1 + f)\)

64. The particle is in a uniform electric field \(E\) and a magnetic field \(B\) which are perpendicular to each other. The particle starts from rest at \(t = 0\) and moves with a constant velocity \(v=\frac{1}{2}\) in the direction of the field \(E\). The total time derivative of the dynamical variable \((p + \sqrt{2}x)\) is

1. \(\frac{1}{2}mc\)
2. \(\frac{mc}{\sqrt{2}}\)
3. \(\sqrt{2}mc\)
4. \(\sqrt{2}mc\)

65. Electromagnetic wave of angular frequency \(\omega\) is propagating in a medium in which, over a band of frequencies, the refractive index is \(n(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_c}\right)^2}\), where \(\omega_c\) is a constant. The ratio \(v_p/v_h\) of the group velocity to the phase velocity at \(\omega = \omega_c/2\) is

1. \(3\)
2. \(1/4\)
3. \(2\)
4. \(2\)

66. In an inertial frame, uniform electric and magnetic fields \(E\) and \(B\) are perpendicular to each other and satisfy \(|E|^2 - |B|^2 = 29\) (in suitable units). In another inertial frame, which moves at a constant velocity with respect to the first frame, the magnetic field is \(2\sqrt{2} E\). In the second frame, an electric field consistent with the previous observations is

1. \(\sqrt{2}(1 + f)\)
2. \(7(1 + f)\)
3. \(\frac{7}{3}(1 + f)\)
4. \(7(1 + f)\)
65. A rotating spherical shell of uniform surface charge and mass density has total mass $M$ and charge $Q$. If its angular momentum is $L$, and magnetic moment is $\mu$, then the ratio $g/L$ is

\[
\begin{align*}
&1. \frac{e}{h} \\
&2. \frac{2e}{3h} \\
&3. \frac{2}{3h} \\
&4. \frac{3e}{4h}
\end{align*}
\]

66. The magnetic moment $\mu = L_d p_d - L_s p_s$ is, where $L_d$ and $L_s$ denote, respectively, the components of the angular momentum of the electron and the magnetic moment operators. The expectation $\langle A_x, x \rangle$ of $A_x$ is the $x$ component of the position operator, is

\[
\begin{align*}
&1. -\langle z p_x + y p_y \rangle \\
&2. -\langle z p_x - y p_y \rangle \\
&3. \langle z p_x + y p_y \rangle \\
&4. \langle z p_x - y p_y \rangle
\end{align*}
\]

67. A one-dimensional system is described by the Hamiltonian $H = \frac{p^2}{2m} + \lambda |x|$. The ground state energy varies as a function of $\lambda$ as

\[
\begin{align*}
&1. \frac{1}{\sqrt{\lambda}} \\
&2. \frac{1}{\lambda^{1/2}} \\
&3. \lambda^{1/2} \\
&4. \lambda^{1/4}
\end{align*}
\]

68. If the position of the electron in the ground state of a Hydrogen atom is measured, the probability that it will be found at a distance $r \geq r_0$ (where $r_0$ is Bohr radius) is nearest to

\[
\begin{align*}
&1. 0.91 \\
&2. 0.66 \\
&3. 0.32 \\
&4. 0.13
\end{align*}
\]

69. The angular part $\phi_q\psi$ of the wavefunction in a magnetic field is $|q|$. The expectation $\langle A_x, x \rangle$ of $A_x$ is the $x$ component of the position operator, is

\[
\begin{align*}
&1. \frac{1}{3} \\
&2. 2/3 \\
&3. 1/4 \\
&4. 3/4
\end{align*}
\]

70. A system of spin $1/2$ particles is prepared in the state $|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$. The spin is rotated by an angle of $\theta$, where $\theta$ is the angle of about the $z$-axis. After the rotation, the state of the system is described by

\[
\begin{align*}
&1. \cos \theta |\uparrow\rangle + \sin \theta |\downarrow\rangle \\
&2. \sin \theta |\uparrow\rangle + \cos \theta |\downarrow\rangle \\
&3. \cos \theta |\uparrow\rangle - \sin \theta |\downarrow\rangle \\
&4. \sin \theta |\uparrow\rangle - \cos \theta |\downarrow\rangle
\end{align*}
\]
71. The Debye approximation, the contribution of the phonons to the heat capacity of a two-dimensional solid is proportional to:
1. $T^2$
2. $T^3$
3. $T^{3/2}$
4. $T^{3/4}$

72. A particle bops on a one-dimensional lattice with lattice spacing $a$. The probability of the particle to hop to the neighboring site to its right is $p$, while the corresponding probability to hop to the left is $q = 1 - p$. The root-mean-squared deviation $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ in displacement after $N$ steps, is:
1. $a\sqrt{Npq}$
2. $aN\sqrt{pq}$
3. $2a\sqrt{Npq}$
4. $aN$
74. The input $V_i$ to the following circuit is a square wave as shown in the following figure.

![Circuit Diagram]

Which of the waveforms best describes the output?

1. 

![Waveform 1]

2. 

![Waveform 2]

3. 

![Waveform 3]

4. 

![Waveform 4]

75. आयतन $f_0$ के वायर विद्युत शक्ति आयतन का अनुपात $\frac{f}{f_0}$ पर मापूर्ति किया जाता है। इस्म में से कौन-सा वायर विद्युत आयतन (पेंट्रम) को सबसे अधिक तरह उक्त किया जाता है?

3-B-H
25. The amplitude of a carrier signal of frequency $f_0$ is sinusoidally modulated at a frequency $f'$ or $f_0$. Which of the following graphs best describes its power spectrum?
FOR ROUGH WORK