प्रश्न पत्र

1. कले हिंदी को नामकरण करें। इस प्रश्न पूरा में एक शारीर (20 मान 'A' है, 40 मा
' B, 60 मान 'C' है) दूरबीन तथा (MCD) दिखाएं। इसके बाद मान 'A' में ही हटाइया।
2. मान 'B' 20 वर्गीकरण है, मान 'C' 40 वर्गीकरण है। इस प्रश्न का विवेक करने के बाद मान 'A' के 10, मान 'B' के 25 तथा मान 'C' के 20 प्रश्नों को ज्ञात करें।

3. प्रश्न का अंश से दिया गया है। केवल यह ही मान 'A' और 'B' के मानों के साथ शीर्षक देखकर ही पूरा में है। मान 'C' के लिए नहीं है। इस प्रश्न का अंश से किसी शीर्षक नहीं है।

4. इस प्रश्न पूरा में एक विंड शीर्षक के लिए मान 'A' का नामन्य विवेक करने की कलेक्शन नहीं है।

5. मान 'A' में प्रश्न का विवेक करने के बाद मान 'B' के 4.75 अंश का नामकरण करने में मान 'C' के 3 अंश का नामकरण करने का विवेक करने को नहीं है।

6. मान 'A' का प्रश्न का विवेक करने के बाद मान 'B' का नामकरण करने के बाद मान 'C' का नामकरण करने का विवेक करने को नहीं है।

7. इस प्रश्न का विवेक करने के बाद मान 'C' का नामकरण करने के बाद मान 'A' का नामकरण करने का विवेक करने को नहीं है।

8. मान 'B' का प्रश्न का विवेक करने के बाद मान 'C' का नामकरण करने के बाद मान 'A' का नामकरण करने का विवेक करने को नहीं है।

9. केवल मान 'B' का प्रश्न का विवेक करने के बाद मान 'C' का नामकरण करने के बाद मान 'A' का नामकरण करने का विवेक करने को नहीं है।

10. मान 'B' का प्रश्न का विवेक करने के बाद मान 'C' का नामकरण करने के बाद मान 'A' का नामकरण करने के बाद मान 'B' का नामकरण करने का विवेक करने को नहीं है।

11. मान 'C' का प्रश्न का विवेक करने के बाद मान 'A' का नामकरण करने के बाद मान 'B' का नामकरण करने का विवेक करने को नहीं है।

12. मान 'A' का प्रश्न का विवेक करने के बाद मान 'B' का नामकरण करने के बाद मान 'C' का नामकरण करने का विवेक करने को नहीं है।
INSTRUCTIONS

1. This Test Booklet contains one hundred and twenty (20 Part 'A' + 40 Part 'B' + 60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A', 'B' and 'C' respectively, will be taken up for evaluation.

2. OMR answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the invigilator to change the booklet of the same code. Likewise, check the OMR answer sheet also. Sheets for rough work have been appended to the test booklet.

3. Write your Roll No., Name and Serial Number of this Test Booklet on the OMR Answer sheet in the space provided. Also put your signatures in the space earmarked.

4. You must darken the appropriate cirles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the OMR Answer Sheet. Failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss including rejection of the OMR answer sheet.

5. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @ 0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.

6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have "ONE" or "MORE" correct options. Credit in a question shall be given only on identification of 'ALL' the correct options in Part 'C'.

7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.

8. Candidate should not write anything anywhere except on OMR answer sheet or sheets for rough work.

9. Use of calculator is not permitted.

10. After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the invigilator and retain the carbonized copy for your record.

11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.
भाग/PART - A

1. एक जिन्दगी जीवन (किस्मत) की, जिसमें एक दर्शनीय दिखलाए दो समय के अमीय विकसित है, की विकसित (cm में) विद्या नहीं है। इस के बारे में कैसे (किस्मत) का अभ्यास (cm में) क्या है?

1. 3.2  
2. 3.6  
3. 6.4  
4. 7.2

2. A boy throws a ball with a speed $v$ at a vehicle that is approaching him with a speed $V$. After bouncing, the ball hits the boy with a speed 1. $v$  
2. $v + V$  
3. $v + 2V$  
4. $v + 4V$

3. चार शिक्षक अपने घर के घंटे पर थे। उन्होंने निर्णय किया कि उन्हें सबसे बड़ी घटना का एक अभिलक्षण दुकान जाना चाहिए। बाँयु, कुटुंबीया से तीन महीने बड़ा है जो कि अपने से तीन महीने छोटा है। देखें से, कुटुंबीया से एक महीने बड़ी है। दो ज्ञातीय एक अभिलक्षण दुकान जाना चाहिए?

1. बाँयु  
2. देखें से  
3. बाँयु  
4. कुटुंबीया

4. चार दोस्त एक घर में भेजते रहते हैं। उन्होंने एक दोस्त को उठाने का आदेश दिया। उन्हें अपने पास के साथ बच्चे के साथ तक की दवाई नहीं है। विषमताएं, दोस्त ने इस दिन जल्दी जाना कि दोस्त में भेजते थे। दोस्त के बीच हो जा सकती है नहीं?

1. बाँयु  
2. देखें से  
3. बाँयु  
4. कुटुंबीया
1. विद्युत्र नहीं बढ़ता है
2. $\frac{dx}{dt}$ से अपर उठता है
3. $\frac{dy}{dt}$ से अपर उठता है
4. $\frac{dz}{dt}$ से अपर उठता है

4. A funnel is connected to a cylindrical vessel of cross sectional area $A$ as shown, to make an interconnected system of vessels. Water is poured in the cylinder such that the height of water in the funnel is $h$ as shown. If the level of water in the cylindrical vessel is pushed down by a distance $x < l$, the level of water in the funnel:

![Diagram of funnel and cylindrical vessel](image)

1. remains unchanged
2. rises by $\frac{dx}{A}$
3. rises by $\frac{dx}{A}$
4. rises by $\frac{dx}{A}$

5. सात छात्रों के अंक (30 अंक में से) एक परीक्षा में 4, 15, 6, 7, 5, a, तथा b है। यदि $a > 70$
का मूल्यांक है, तब $b$ एक अलग अंक है।
इस समूह में अंक एक है जिसे ('Range')
(अधिकतम अंक - न्यूनतम अंक) में समाप्त
किया गया है?
1. 25
to 26
2. 27
to 28
3. 29
to 29

5. Marks (out of 30) of seven students in an examination are 4, 15, 6, 7, 5, a and b, where $a > 70$ is a multiple of 4 and $b$ is a prime. What is the maximum possible value of the range of marks (i.e. maximum mark - minimum mark)?
1. 25
to 26
2. 27
to 28
3. 29
to 29

6. यदि वृत्त A और B एक बिंदु से निर्देशित दिशाओं में घूमते हैं। A की गति $B$ से दूरी है। B की गति $1$ km/h है।
वंदे 2 km पार करने के बाद A वापस मुक्त B की तरफ घूमता है। अतः A प्राथमिक बिंदु से विस्तारित कर बैठी है। टांग A से कितनी दूरी पर B से अगले घटित नहीं?
1. 2 km
2. 4 km
3. 6 km
4. 8 km

6. Two persons A and B start walking in opposite directions from a point. A travels twice as fast as B. The speed at which B travels is 1 km/h. If A travels 2 km and turns back and starts walking towards B, at what distance from the starting point will A cross B?
1. 2 km
2. 4 km
3. 6 km
4. 8 km

7. एक व्यक्ति कार से बराबर से आतंकवादी
एक स्थान 60 km/h कपड़े पहने है।
स्थान से आतंकवादी की दूरी 2 km है।
आतंकवादी की घट लेते हैं एक
विशिष्ट व्यक्ति से 60 km/h की गति.
क्योंकि यह हुआ सबसे बड़े रूप से रिसङ
से यहाँ के 60 km/h की गति के
लक्षण को पहनें?
1. 60 km/h
2. 90 km/h
3. 120 km/h

7. A person wanted to travel from Charbagh to Alambagh with an average speed of 60 km/h by car. The distance between Charbagh and Alambagh is 2 km. Due to heavy traffic, he could travel at 30 km/h for the first kilometre of his journey. What should his speed be for the remaining journey to achieve his average speed target of 60 km/h?
1. Cannot achieve his target with any
finite speed.
2. 60 km/h
3. 90 km/h
4. 120 km/h
8. The average rainfall over a given place during the three-year period of 2003-2005 was 65 cm. During the three-year period 2002-2004 the average rainfall was 63 cm. The actual rainfall during 2005 was 60 cm. What was the rainfall in 2002?
1. 55 cm
2. 60 cm
3. 54 cm
4. 53 cm

9. The average rainfall over a given place during the three-year period of 2003-2005 was 65 cm. During the three-year period 2002-2004 the average rainfall was 63 cm. The actual rainfall during 2005 was 60 cm. What was the rainfall in 2002?
1. 55 cm
2. 60 cm
3. 54 cm
4. 53 cm

10. After 6 g of fuel is completely burnt in an atmosphere of 40 g of oxygen, the percentage oxygen left is:
1. 30
2. 60
3. 40
4. 20

11. The average rainfall over a given place during the three-year period of 2003-2005 was 65 cm. During the three-year period 2002-2004 the average rainfall was 63 cm. The actual rainfall during 2005 was 60 cm. What was the rainfall in 2002?
1. 55 cm
2. 60 cm
3. 54 cm
4. 53 cm

13. What fraction of the equilateral triangle shown below with three identical sectors of a circle is shaded?

1. \(1 - \frac{\pi}{\sqrt{3}}\)
2. \(\frac{\pi}{\sqrt{3}}\)
3. \(\frac{2\pi}{\sqrt{3}}\)
4. \(4 - \frac{\sqrt{3}}{2}\)

12. How many different salads can be made from cucumber, tomatoes, onions, beetroot and carrots?
1. 16
2. 28
3. 31
4. 32

13. What fraction of the equilateral triangle shown below with three identical sectors of a circle is shaded?

1. \(1 - \frac{\pi}{\sqrt{3}}\)
2. \(\frac{\pi}{\sqrt{3}}\)
3. \(\frac{2\pi}{\sqrt{3}}\)
4. \(4 - \frac{\sqrt{3}}{2}\)
13. A bottle of perfume is opened and a person at a distance of 10 m gets the smell after 10 seconds. The time taken for a person 20 m away to get the smell is about
1. 2s 2. 4s 3. 14s 4. 80s

14. A mineral contains a cubic and a spherical cavity. The length of the side of the cube is the same as the diameter of the sphere. If the cubic cavity is half filled with a liquid and the spherical cavity is completely filled with liquid, what is the approximate ratio of the volume of liquid in the cubic cavity to that in the spherical cavity?
1. 2:1 2. 1:1 3. 1:2 4. 1:4

15. Of the 6 unbiased coins, 5 are tossed independently and they all result in heads. If the 6th is now independently tossed, the probability of getting head is
1. 1 2. 0 3. 1/2 4. 1/6

16. What could the fourth figure in the sequence be?

17. The average age of A, B, and C, whose ages are integers $x, y$ and $z$ respectively ($x \leq y \leq z$), is 30. If the age of B is exactly 5 more than that of A, what is the minimum possible value of $x$?
1. 31 2. 33 3. 35 4. 37

18. The statement is: “$\forall x \forall y (x \leq y)$”.

19. $A$, $B$ and $C$ are three points in a plane. $D$ is a point on the line segment $AB$. If $D$ is the mid-point of $AB$, then $AD = BD$.
18. Percentage-wise distribution of all science students in a university is given in the pie-diagram. The bar chart shows the distribution of physics students in different sub-areas, where a student takes one and only one sub-area. What percentage of the total science students is girls studying quantum mechanics?

19. What is the total number of parallelograms in the given diagram?

20. In a city, there are three schools (A, B, and C) that have different grades. The table shows the percentage of students in each grade. The question is about the number of students in each grade. Which school has the highest number of students in grade X?

---

1. 10  2. 1
3. 0.2  4. 2

1. 27  2. 24
3. 22  4. 14

1. 70  2. 15
3. 20  4. 15

1. V  2. X
3. Y
4. X and Y together
20. Election results of a city, which contains 3 segments (A, B and C) are given in the table. Percentage votes obtained by parties X, Y and Z are also shown. Which party won the election?

<table>
<thead>
<tr>
<th>Segment</th>
<th>Total Voters</th>
<th>% of Voting</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,00,000</td>
<td>60</td>
<td>30</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>1,50,000</td>
<td>70</td>
<td>40</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>2,00,000</td>
<td>80</td>
<td>30</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

1. Y
2. X
3. Z
4. It was a tie between X and Y

22. Define \( f(x) = \frac{1}{x} \) for \( x > 0 \). Then \( f \) is uniformly continuous
   1. on \([0, \infty)\)
   2. on \([r, \infty)\) for any \( r > 0 \)
   3. on \((0, r)\) for any \( r > 0 \)
   4. only on intervals of the form \([a, b]\) for \( 0 < a < b < \infty \)

23. \( R^2 \) की सिद्धांतीय दो उपसमुच्चय \( W_1 \) तथा \( W_2 \) को इस प्रकार असमान परिभाषित किया जाता है \( W_1 = \{(x, y, z) \in R^3 : x + y + z = 0\} \) तथा \( W_2 = \{(x, y, z) \in R^3 : x - y + z = 0\} \). यदि \( W_1 \) तथा \( W_2 \) इस तरह उपसमुच्चय परिभाषि के फूड़े हैं तो
   \( W_1 \cap W_2 = \text{समुच्चय (0,1,0)} \)
   \( W_1 \cup W_2 = \text{समुच्चय (0,0,1)} \)
   \( W_1 \cap W_2 = \text{समुच्चय (0,1,0)} \)
   \( W_1 \cup W_2 = \text{समुच्चय (0,0,1)} \)

24. \( C = \left[ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right] \) को \( R^2 \) का अभावी भाग तथा \( T : R^2 \rightarrow R^2 \) को \( T(x) = \begin{bmatrix} x + y \\ x - y \end{bmatrix} \) से परिभाषित करते हैं. \( A \) का \( T[C] \) के दिये से लिये \( T \) का अभावी भाग \( T[C] \) से दिये हए \( C \) में से कोना सबसे अभावी भाग है?
1. \( T(C) = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix} \)
2. \( T(C) = \begin{bmatrix} 3 & -2 \\ -3 & 1 \end{bmatrix} \)
3. \( T(C) = \begin{bmatrix} 3 & -2 \\ 3 & 1 \end{bmatrix} \)
4. \( T(C) = \begin{bmatrix} -3 & -2 \\ -3 & 1 \end{bmatrix} \)

24. Let \( C = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \) be a basis of \( \mathbb{R}^2 \) and 
\( T : \mathbb{R}^2 \to \mathbb{R}^2 \) be defined by 
\( T(x) = \begin{bmatrix} x + y \\ x - 2y \end{bmatrix} \). If \( T(C) \) represents the matrix of 
\( T \) with respect to the basis \( C \) then which of the following is true?
1. \( T(C) = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix} \)
2. \( T(C) = \begin{bmatrix} -2 & 3 \\ 1 & 3 \end{bmatrix} \)
3. \( T(C) = \begin{bmatrix} 3 & -2 \\ 3 & 1 \end{bmatrix} \)
4. \( T(C) = \begin{bmatrix} -3 & -2 \\ -3 & 1 \end{bmatrix} \)

25. Let \( W_1 = \{(u, v, w, x) \in \mathbb{R}^4 \mid u + v + w = 0, 
2v + x = 0, 2u + 2w - x = 0\} \). Let \( W_2 = \{(u, v, w, x) \in \mathbb{R}^4 \mid u + w + x = 0, 
u + w - 2x = 0, v - x = 0\} \). True or False?
1. \( \dim(W_1) = 2 \)
2. \( \dim(W_2) = 3 \)
3. \( \dim(W_1 \cap W_2) = 1 \)
4. \( \dim(W_1 + W_2) = 3 \)

26. Let \( A \) be an \( n \times n \) complex matrix. 
Assume that \( A \) is self-adjoint and let \( B \) 
define the inverse of \( A + iI \). Then all 
eigenvalues of \( (A - I^2)B \) are
1. purely imaginary
2. of modulus one
3. real
4. of modulus less than one

27. Let \( C^* \) be the conjugate transpose of \( C \). 
A matrix \( A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \). 
Which of the following is true?
1. \( \text{Trace}(A) = \sum_{i=1}^{n} a_{ii} \)
2. \( \text{Trace}(A^T) = \sum_{i=1}^{n} a_{ii} \)
3. \( \text{Trace}(A^T A) = \sum_{i=1}^{n} a_{ii} \)
4. \( \text{Trace}(A^T A) = n \)

28. Let \( \{u_1, u_2, \ldots, u_n\} \) be an orthonormal 
basis of \( \mathbb{C}^n \) as column vectors. Let 
\( M = \langle \{u_1, u_2, \ldots, u_n\} \rangle \) and 
\( P \) be the diagonal \( k \times k \) matrix with 
diagonal entries \( a_1, a_2, \ldots, a_k \). \( \langle \{u_1, u_2, \ldots, u_n\} \rangle \). 
Which of the following is true?
1. \( \text{Rank}(M^T) = k \) whenever \( a_i = a_j \)
2. \( \text{Trace}(M^T) = \sum_{i=1}^{k} a_i \)
3. \( \text{Rank}(M^T M) = \min\{k, n-k\} \)
4. \( \text{Trace}(M^T + M^T) < n \)

29. Let \( B : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \) be a function \( B(x, y) = \) some 
function that is continuous at \( (0,0) \). Which of the following is true?
1. \( B \) is continuous everywhere
2. \( B \) is differentiable at \( (0,0) \)
3. \( B \) is differentiable at \( (0,0) \) but not continuous
4. \( B \) is continuous but not differentiable at \( (0,0) \)
28. Let \( B : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \) be the function \( B(a, b) = ab \).
Which of the following is true?
1. \( B \) is a linear transformation
2. \( B \) is a positive definite bilinear form
3. \( B \) is symmetric but not positive definite
4. \( B \) is neither linear nor bilinear

29. निम्न अवस्था परिभाषित प्रतिविद्यमान मूल संकेत \( f : \mathbb{Q} \to \mathbb{R} \) के लिए
(i) \( f(0) = 0 \)
(ii) \( f(r) = \frac{1}{r} \), जहाँ \( r \in \mathbb{Q} \) जबकि \( p \in \mathbb{Z}, q \in \mathbb{N} \) तथा \( \gcd(p, q) = 1 \).

30. यदि \( x \) का क्षणिक संख्या के लिए \( |x| < 1 \), तो \( \frac{1}{1-x} \) का क्षणिक संख्या है।
1. \( \frac{1}{1-x} \) का क्षणिक संख्या है
2. \( \frac{1}{1-x} \) का क्षणिक संख्या है
3. \( \frac{1}{1-x} \) का क्षणिक संख्या है
4. \( \frac{1}{1-x} \) का क्षणिक संख्या है

31. Suppose that \( \{x_n\} \) is a sequence of real numbers satisfying the following. For every \( \epsilon > 0 \), there exists \( n_0 \) such that
\[ |x_{n+1} - x_n| < \epsilon \quad \forall \quad n \geq n_0 \]
The sequence \( \{x_n\} \) is
1. bounded but not necessarily Cauchy
2. Cauchy but not necessarily bounded
3. convergent
4. not necessarily bounded

32. \[ A(n) = \int_{n}^{n+1} \frac{1}{x} \, dx \text{ जबकि } n \geq 1. \]
\( x \in \mathbb{R} \) के लिए निम्न लिखित संदर्भ हैं:
1. \( L = 0 \) वर्तमान \( c > 3 \)
2. \( L = 1 \) वर्तमान \( c = 3 \)
3. \( L = 2 \) वर्तमान \( c = 3 \)
4. \( L = \infty \) वर्तमान \( 0 < c < 3 \)

33. Let \( A(n) = \int_{n}^{n+1} \frac{1}{x} \, dx \) for \( n \geq 1 \).
For \( c \in \mathbb{R} \) let \( \lim_{n \to \infty} n^c A(n) = L \).
Then
1. \( L = 0 \) if \( c > 3 \)
2. \( L = 1 \) if \( c = 3 \)
3. \( L = 2 \) if \( c = 3 \)
4. \( L = \infty \) if \( 0 < c < 3 \)
33. Consider the polynomials $p(x)$ and $q(x)$ in the complex variable $z$ and let

\[ l_{p,q} = \oint_{C} p(z)q(\overline{z}) \, dz \]

where $C$ denotes the closed contour $y(t) = e^{it}, 0 \leq t \leq 2\pi$. Then

1. $l_{n,n} = 0$ for all positive integers $n$.
2. $l_{n,n} = 2\pi i$ for all positive integers $n$.
3. $l_{n,1} = 0$ for all polynomials $p$.
4. $l_{p,q} = p(0)q(0)$ for all polynomials $p,q$.

34. Let $\gamma(t) = 3\cos t, 0 \leq t \leq 2\pi$ be the positively oriented circle of radius 3 centered at the origin. The value of $\lambda$ for which

\[ \oint_{\gamma} \frac{1}{z^2 - 5z + 4} \, dz = \oint_{\gamma} \frac{1}{z^2 - 5z + 4} \, dz \]

is

1. $\lambda = -1/3$
2. $\lambda = 0$
3. $\lambda = 1/3$
4. $\lambda = 1$

35. The number of group homomorphisms from the alternating group $A_5$ to the symmetric group $S_4$ is:

1. 1
2. 12
3. 20
4. 4

36. Let $p \geq 23$ be a prime number such that the decimal expansion (base 10) of $\frac{1}{p}$ is periodic with period $p - 1$ (that is, $\frac{1}{p} = 0.a_1a_2 \ldots a_{p-1} \overline{a_1a_2 \ldots a_{p-1}}$) with $a_i \in \{0,1, \ldots, 9\}$ for all $i$ and for any $m, 1 \leq m < p - 1, a_i \neq 0$. Let $\mathbb{Z}/p\mathbb{Z}^*$ denote the multiplicative group of integers modulo $p$. Then which of the following is correct?

1. The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a proper divisor of $(p - 1)$.
2. The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is $\frac{p-1}{2}$.
3. The element $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a generator of the group $(\mathbb{Z}/p\mathbb{Z})^*$.
4. The group $(\mathbb{Z}/p\mathbb{Z})^*$ is cyclic but not generated by the element 10.
37. Given integers $a$ and $b$, let $N_{a,b}$ denote the number of positive integers $k < 100$ such that $k \equiv a \pmod{9}$ and $k \equiv b \pmod{11}$. Then which of the following statements is correct?

1. $N_{a,b} = 1$ for all integers $a$ and $b$.
2. There exist integers $a$ and $b$ satisfying $N_{a,b} > 1$.
3. There exist integers $a$ and $b$ satisfying $N_{a,b} = 0$.
4. There exist integers $a$ and $b$ satisfying $N_{a,b} = 0$ and there exist integers $c$ and $d$ satisfying $N_{c,d} > 1$.

38. Let $X$ be a topological space and $U$ be a proper dense open subset of $X$. Pick the correct statement from the following:

1. If $X$ is connected then $U$ is connected.
2. If $X$ is compact then $U$ is compact.
3. If $X \setminus U$ is compact then $X$ is compact.
4. If $X \setminus U$ is compact, then $X$ is compact.

39. Let $R$ denote the radius of convergence of the power series

$$\sum_{k=1}^{\infty} k x^k.$$

Then

1. $R > 0$ and the series is convergent on $[-R, R]$.
2. $R > 0$ and the series converges at $x = -R$ but does not converge at $x = R$.
3. $R > 0$ and the series does not converge outside $(-R, R)$.
4. $R = 0$.

40. Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let

$$\text{Image}(f) = \{w \in \mathbb{C} : 3 \pi \in \mathbb{C} \text{ such that } f(z) = w\}.$$

Then

1. $\text{Image}(f)$ is an open set.
2. $\text{Image}(f)$ is not connected.
3. $\text{Image}(f)$ is not closed.
4. $\text{Image}(f)$ is a non-constant entire function.

41. Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let

$$\text{Image}(f) = \{w \in \mathbb{C} : 3 \pi \in \mathbb{C} \text{ such that } f(z) = w\}.$$

Then

1. The interior of $\text{Image}(f)$ is empty.
2. $\text{Image}(f)$ intersects every line passing through the origin.
3. There exists a disc in the complex plane which is disjoint from $\text{Image}(f)$.
4. $\text{Image}(f)$ contains all its limit points.
41. Given that $u(x, t)$ is a function which satisfies the partial differential equation (PDE):

$$u_{tt} - u_{xx} = e^x + 6t, \quad x \in \mathbb{R}, \quad t > 0$$

with the initial conditions:

$$u(x, 0) = \sin(x), \quad u_t(x, 0) = 0$$

for every $x \in \mathbb{R}$.

Then the value of $\lim_{t \to 0^+} u(x, t)$ equals:

1. $e$
2. $\pi$
3. $\frac{1}{2}$
4. $1$

43. Given that $f(x)$ is a function of unknown degree satisfying the conditions:

$$f(2) = 2, \quad f(7) = 12, \quad f(16) = 32$$

then $x^2$ is a root of $f(x)$.

44. Given the functional:

$$J[y] = \int \left( (1 - y'^2)^2 dx \right)$$

Find the value of $\lim_{t \to 0^+} u(x, t)$. The possible values are:

1. $e$
2. $\pi$
3. $\frac{1}{2}$
4. $1$

44. Consider the functional:

$$J[y] = \int \left( (1 - y'^2)^2 dx \right)$$

defined on $y \in C^2[0, 2]$, which is piecewise $C^1$ and $y(2) = y(0) = 0$. Let $y_0$ be a minimizer of the above functional. Then $y_0$ has
1. a unique corner point
2. two corner points
3. more than two corner points
4. no corner points

45. \[ \int (1 - x^2 + t^2) \phi(t) dt = \frac{x^2}{2} \]
where \( \phi \) is given by \( \phi(v^2) \) and \( V \) is constant
1. \( \sqrt{2}e^{-2} \)
2. \( \sqrt{2}e^{-3} \)
3. \( \sqrt{2}e^{5/2} \)
4. \( \sqrt{2}e^{a^4} \)

46. \( \frac{\phi}{ \phi(v^2)} \) is the solution of
\[ \int (1 - x^2 + t^2) \phi(t) dt = \frac{x^2}{2} \]
then \( \phi(v^2) \) is equal to
1. \( \sqrt{2}e^{-2} \)
2. \( \sqrt{2}e^{-3} \)
3. \( \sqrt{2}e^{5/2} \)
4. \( \sqrt{2}e^{a^4} \)

47. यदि, \( \cos z \) \( y'' + (\sin z) y' - (1 + e^{-x^2}) y = 0 \)
\( \forall x \in \mathbb{R} \)
के दो हल \( y_1(x) \) तथा \( y_2(x) \) हैं, जिसके
\( y_1(0) = \sqrt{2}, y_2(0) = 1 \), \( y_1(0) = -\sqrt{2}, y_2(0) = 2 \)
तब \( x = \frac{\pi}{4} \) पर \( y_1(x) \) तथा \( y_2(x) \) का
tरेखांकन है
1. \( 3\sqrt{2} \)
2. \( 6 \)
3. \( 3 \)
4. \( -3\sqrt{2} \)

48. दिये गये संख्य
\[ x(t) = x - at + y \]
\[ y(t) = 2x - 2y - 3y \cos (y^2) \]
का आकारों दिनु (0,0) है.
1. \( \text{x-अर्ध्यलग दिनु} \)
2. \( \text{धारा सरिता दिनु} \)
3. \( \text{धारा सरिता दिनु} \)
4. \( \text{सिंह आर्ड्यलग} \)
48. The critical point $(0,0)$ for the system

\[ x'(t) = -2x + y^2 \sin(x) \]
\[ y'(t) = 2x - 2y - 3y \cos(y^2) \]

is a

1. stable spiral point
2. unstable spiral point
3. saddle point
4. stable node

**Unit-4**

49. परिस्थिति प्रतिफलक \( T \) का उपयोग करते हुए परिस्थितियाँ \( H_0 \) की भर्ती परिस्थिति \( H_1 \) के माध्यम से परिस्थिति परिस्थिति \( H_0 \) का समान्तर नहीं करती है यदि \( T \) पर मान आधिक है। इसलिए \( \widehat{T} \) (sample) के आधार पर परिस्थिति प्रतिफलक का \( p \)-मान 0.05 सत्त्व से अधिक है यदि यह मान \( H_0 \) के अधिक होना \( H_1 \) के अधिक \( \beta \) फलन का समान्तर \( 0,0,1 \) है। \( H_0 \) के अधिक \( T \) के वेंटन ये 10 स्क्रूवेज शीट के साथ तेज़ बहन होती है, \( p \)-मान होगा

1. 0.05
2. \( < 0.05 - \frac{1}{120} \)
3. \( 0.05 - \frac{1}{100} \)
4. \( > 0.05 \)

50. संगणक \((x_1,y_1),(x_2,y_2), \ldots, (x_n,y_n)\) की दीवार संख्या बहन के \( n \) स्क्रूवेज देखता है। \( n \) आवश्यक आधार पर माने कि \( r \) संधिक युग्मक आवश्यक लिखी संधिक युग्मक \( r \) का संधिक युग्मक आवश्यक है। विद्युत \( m \) से फिल्टर का व्यवस्था करने का \( p \)-मान 0.05 पर समान्तर \( H_0 \) का समान्तर \( H_1 \) के साथ \( T \) का \( H_0 \) के अधिक \( \beta \) फलन 10 स्क्रूवेज शीट के साथ तेज़ बहन होती है, \( p \)-मान होगा

1. 0.05
2. \( < 0.05 - \frac{1}{120} \)
3. \( 0.05 - \frac{1}{100} \)
4. \( > 0.05 \)

51. \( \widehat{T} \) के दीवार मोडल \( \gamma_1 = \theta_1 + \theta_2 + \gamma_i \) (जहाँ \( i = 1,2 \) तक \( \gamma_i = \theta_1 - \theta_2 + \gamma_i \) (जहाँ \( i = 3,4 \)) पर विचार के अनुसार जाने कि आवश्यक \( \gamma_i \) वस्तुता है तथा \( i = 1,2,3,4 \) के लिए \( E(\gamma_i) = 0, Var(\gamma_i) = \sigma^2 > 0 \) साथ \( \theta_1, \theta_2, \theta_3 \in \mathbb{R} \) के प्राचीन जोड़कर गणना आवश्यक है?

1. \( \theta_1 + \theta_2 \)
2. \( \theta_1 - \theta_2 \)
3. \( \theta_2 + \theta_3 \)
4. \( \theta_1 + \theta_2 + \theta_3 \)

52. \( \theta_1 \) एक लाइनर मॉडल \( Y = \theta_1 + \theta_2 + \gamma_i \) के लिए लाइनर \( i = 1,2,3,4 \) के \( \gamma_i \) के अलग \( \gamma_i \) के प्राचीन गणना आवश्यक है?

1. \( \theta_1 + \theta_2 \)
2. \( \theta_1 - \theta_2 \)
3. \( \gamma_1 + \gamma_2 \)
4. \( \theta_1 + \theta_2 + \theta_3 \)
51. \( (X - N_0(0, I)) \) என்கிறோம் \( A_{\mathcal{R}} \) என்றும் வரையனாட்டுப் பாரம்பரியாகும் விளக்கம் தெரியும் கிள்ளின் \((A) = k < p \) தேன். \( \mathcal{R} \) ஐ நிலையிலியைச் சேர்ந்த பொருளான குறிப்பிட்டு படுத்தியுள்ளார்:
\[ \begin{align*}
1. \quad & X'X - \beta' \beta \quad \text{\( X'X \)} \\
2. \quad & X'X - \beta' \beta \quad \text{\( X'X \)} \\
3. \quad & X'X - \beta' \beta \quad \text{\( X'X \)} \\
4. \quad & X'X - \beta' \beta \quad \text{\( X'X \)} \\
\end{align*} \]

52. \( (X \sim N_p(0, I)) \) செய்யப்பட்டு \( D_{\mathcal{R}} \) என்பது ஒரு முறையும் மஞ்சள்ளியின் வரிசைச் சார்ந்து அடிப்படையில், \( k < p \) என்றும் மீது இருப்பதாக வரையறைப்பட்டுள்ளார்.

53. \textit{PPSM} \textit{R} படிவு தொகுதியின் குறைத்தொகுதியை \( N(\geq 2) \) க் ஸ்பெய்சின் மையானுடைய இருபாதிக்கு பொருந்தும் வரிசையில் \( \mathcal{R} \) என்றும் ஒரு செய்யப்பட்டுள்ள வரிசையில் \( \mathbf{P}_R \) க்குத் தகுந்து, \( 0 < p \), \( 1 \quad \text{\( \begin{align*}
1. & \quad 1 - p^2 - p + (p_1 + p_2 + p_3) \\
2. & \quad 1 - (p_1 + p_2 + p_3) \\
3. & \quad 1 - (1 - p_1) + (1 - p_2) - (p_1 + p_2) \\
4. & \quad 1 - (1 - p_1) + (1 - p_2) + (1 - p_3) \\
\end{align*} \)}

53. A sample of size \( n \geq 2 \) is drawn from a population of \( N(\geq 2) \) units using PPSWR sampling scheme, where \( p_i \) is the probability of selecting \( i^{th} \) unit in a draw, \( 0 < p_i < 1 \) \( \forall i = 1, \ldots, N \), and \( \sum_{i=1}^{N} p_i = 1 \). Then the inclusion probability \( p_{ij} \) is
\[ \begin{align*}
1. & \quad 1 - p^2 - p + (p_1 + p_2 + p_3) \\
2. & \quad 1 - (p_1 + p_2 + p_3) \\
3. & \quad 1 - (1 - p_1) + (1 - p_2) - (p_1 + p_2) \\
4. & \quad 1 - (1 - p_1) + (1 - p_2) + (1 - p_3) \\
\end{align*} \]

54. \( (X \sim N_p(0, I)) \) என்றும் \( A^{(R)} \) என்றும் வரையனாட்டுப் பாரம்பரியாகும் விளக்கம் தெரியும் கிள்ளின் \((A) = k < p \) தேன். \( \mathcal{R} \) ஐ நிலையிலியைச் சேர்ந்த பொருளான குறிப்பிட்டு படுத்தியுள்ளார்:
\[ \begin{align*}
1. & \quad ABC \\
2. & \quad ABD \\
3. & \quad BCD \\
4. & \quad ABCD \\
\end{align*} \]

55. In a \( 2^k \) experiment with two blocks and factors \( A, B, C \) and \( D \), one block contains the following treatment combinations:
\[ \begin{align*}
1. & \quad ABC \\
2. & \quad ABD \\
3. & \quad BCD \\
4. & \quad ABCD \\
\end{align*} \]

55. \( \text{Poisson} \) புள்ளி \( \text{PPSM} \textit{R} \) என்றும் ஒரு செய்யப்பட்டு எண்ணிக்கை வரையறைப்பட்டுள்ள வரிசையில் \( \mathcal{R} \) என்றும் ஒரு செய்யப்பட்டுள்ள வரிசையில் \( \mathbf{P}_R \) க்குத் தகுந்து, \( 0 < p \), \( 1 \quad \text{\( \begin{align*}
1. & \quad 1 - p^2 - p + (p_1 + p_2 + p_3) \\
2. & \quad 1 - (p_1 + p_2 + p_3) \\
3. & \quad 1 - (1 - p_1) + (1 - p_2) - (p_1 + p_2) \\
4. & \quad 1 - (1 - p_1) + (1 - p_2) + (1 - p_3) \\
\end{align*} \)}

55. In an airport, domestic passengers and international passengers arrive independently according to Poisson processes with rates 100 and 70 per hour, respectively. If it is given that the total number of passengers (domestic and international) arriving in that airport between 9:00 AM and 11:00 AM on a particular day was 520, then what is the conditional distribution of the number of domestic passengers arriving in this period?
\[ \begin{align*}
1. & \quad \text{Poisson} (200) \\
2. & \quad \text{Poisson} (100) \\
3. & \quad \text{Binomial} \left( 520, \frac{10}{17} \right) \\
4. & \quad \text{Binomial} \left( 520, \frac{10}{17} \right) \\
\end{align*} \]
56. यदि \( X \geq 0 \) (0, \( \mathcal{F} \)) पर एक बाहिरिक घर है जिसके लिए \( \mathbb{E}(X) = 1 \) है। यदि \( A \in \mathcal{F} \) एक प्रारंभिक घर है जिसके लिए \( 0 < P(A) < 1 \) है, तब \( X \) पर स्थिति घर \\( \mathcal{F} \\) की आवश्यकता कटा है?

1. \( Q(B) = P(A \cap B) \quad \forall B \in \mathcal{F} \\)
2. \( Q(B) = P(A \cup B) \quad \forall B \in \mathcal{F} \\)
3. \( Q(B) = E(X_B) \quad \forall B \in \mathcal{F} \\)
4. \( Q(B) = \left\{ \begin{array}{ll} P(B) & \text{if } P(B) > 0 \\ 0 & \text{if } P(B) = 0 \end{array} \right. \)

58. Suppose \( \{X_n\} \) is a Markov Chain with 3 states and transition probability matrix

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Then which of the following statements is true?
1. \( \{X_n\} \) is irreducible
2. \( \{X_n\} \) is recurrent
3. \( \{X_n\} \) does not admit a stationary probability distribution
4. \( \{X_n\} \) has an absorbing state

59. यदि \( X \sim \text{Cauchy}(0,1) \), तब \( \frac{1-x}{1+2x} \) बन्द है?

1. Uniform (0,1)
2. Normal (0,1)
3. ज्ञानशास्त्र (0,1)
4. Cauchy (0,1)

59. Suppose \( X \sim \text{Cauchy}(0,1) \). Then the distribution of \( \frac{X}{\sqrt{X}} \) is

1. Uniform (0,1)
2. Normal (0,1)
3. Double exponential (0,1)
4. Cauchy (0,1)

60. दिनांक 0.5, 0.7, 0.9, 1.2, 1.6, 1.8, 1.4, 0.8, 1.62 दिए गए हैं जो कि समान बन्द (0 - 2, 0 + 0.8) व्युत्क्रम \( -\infty < \theta < \infty \) पर समान बन्द है।

\( \theta \) के लिए जिन में से कौन सा समान बन्द अबस्लाय कारण है?

1. 0.7 
2. 0.9 
3. 1.1 
4. 1.3

60. Given the observations 0.8, 0.71, 0.9, 1.2, 1.6, 1.4, 0.8, 1.62 from the uniform distribution on \( (0 - 0.2, 0 + 0.8) \) with \( -\infty < \theta < \infty \), which of the following is a maximum likelihood estimate for \( \theta \)?

1. 0.7 
2. 0.9 
3. 1.1 
4. 1.3

4.C
भाग/PART - C

Unit-1

61. निम्न के प्रजीज्ञ 1 संख्या है कि  
\[ f(x, y) = 2x^3 + 3xy^2 - 15x - 12y, x + y \]  
की ग्राहित सास्त्र सार। हाँ, \( S = \{(x, y) \in \mathbb{R}^2 : f(x) \leq 0\} \)  
1. \( S = \mathbb{R}^2 \)  
2. \( S \) संयोजन \( \mathbb{R}^2 \) से समान है  
3. \( S \) संयोजन \( \mathbb{R}^2 \) से समान है  
4. \( \mathbb{R}^3 \) संयोजन समान है

62. \( X = \mathbb{N} \) पर \( \mathbb{R}^2 \) की किसी भूमिका का अनुवर्तन करें।  
\( X \) पर मैट्रिक्स \( d_1, d_2 \) पर \( \mathbb{N} \) के नियामक की, जहाँ  
\[ d_1(m, n) = |m - n|, m, n \in X \]  
\[ d_2(m, n) = \frac{|m - n|}{m, n \in X} \]  
यदि \( X_1, X_2 \) के अनुवर्तन, \( X_1, X_2 \) के नियामक हो, तो  
1. \( X_1 \) पूर्ण है  
2. \( X_2 \) पूर्ण है  
3. \( X_1 \) संयोजन \( \mathbb{N} \) से समान है  
4. \( X_2 \) संयोजन \( \mathbb{N} \) से समान है

63. \( \mathbb{R}^n \to \mathbb{R}^n \) के अनुसार \( \mathbb{R}^n \) की संख्या को \( T = I_n \) को संयोजन करने वाली त्रिभुज प्रतिष्ठा संख्या है। \( T^3 = I_n \) को संयोजन करने वाली त्रिभुज प्रतिष्ठा संख्या है। तब निम्न में से कोना से कथन सच है?
1. \( T \) संयोजनीय है  
2. \( T \) संयोजनीय नहीं है  
3. \( T \) का सारणांक \( \mathbb{K} \) संयोजनीय संख्या है  
4. \( T^3 = I_n \)

64. \( f: \mathbb{R}^2 \to \mathbb{R}^2 \) be a linear map that satisfies \( T^3 = T - I_n \). Then which of the following are true?
1. \( T \) is invertible  
2. \( T \) is not invertible  
3. \( T \) has a real eigenvalue  
4. \( T^3 = I_n \)

65. \[ A = \begin{bmatrix} 2 & 3 & 2 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 1 & 1 \end{bmatrix} \]  
\[ b = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \]  
तब निम्न में से कोना से कथन सच है?
1. \( \text{दोनों} \ \text{फल} \ MX = b_1 \text{लगा} \ MX = b_2 \text{संगम है} \)  
2. \( \text{दोनों} \ \text{फल} \ MX = b_1 \text{लगा} \ MX = b_2 \text{संगम है} \)  
3. \( \text{दोनों} \ \text{फल} \ MX = b_1 - b_2 \text{संगम है} \)  
4. \( \text{दोनों} \ \text{फल} \ MX = b_1 - b_2 \text{संगम है} \)

66. Let \( \mathcal{M} = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \)  
\[ b = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \]  
\[ b_2 = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \]  
Then which of the following are true?
1. both systems \( MX = b_1 \) and \( MX = b_2 \) are inconsistent  
2. both systems \( MX = b_1 \) and \( MX = b_2 \) are consistent  
3. the system \( MX = b_1 - b_2 \) is consistent  
4. the system \( MX = b_1 - b_2 \) is inconsistent

4-C-H
65. \( M = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \)

66. Let \( M = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \). Given that 1 is an eigenvalue of \( M \), then which among the following are correct?

1. The minimal polynomial of \( M \) is \((X - 1)(X + 4)\).
2. The minimal polynomial of \( M \) is \((X - 1)^2(X + 4)\).
3. \( M \) is not diagonalizable.
4. \( M^{-1} = \frac{1}{5} (M + 3I) \).

67. Let \( P(x) = x^3 + 2x^2 + x + 1 \) be a polynomial.

68. Let \( M \) be an \( n \times n \) matrix with characteristic polynomial \((X - 1)^3\).

69. \( P(x) \) is a polynomial of degree 3 with \( P(1) = 2 \). Find the value of \( P(-1) \).
69. Let $R^2 \times R^2$ be a vector space. Define the function $\varphi: R^2 \times R^2 \to R^2$ as follows: $\varphi(x, y) = (x_1, y_1)$ for all $(x, y) \in R^2$. For $u, v \in R^2$, let $W = \{w \in R^2 : B(u, w) = 0\}$, where $B(x, y) = x_1y_1 - x_2y_2 + 3x_1y_2 - 2x_2y_1$. Then $W$ is a subspace of $R^2$.

70. Let $\mathbb{R}^2$ be the set of all points $(x, y)$ in $R^2$. Define the function $Q: \mathbb{R}^2 \to \mathbb{R}$ by $Q(x, y) = x^2 + 2xy + y^2$. Consider the quadratic forms $Q_1(x, y) = x^2 + 2xy + y^2$ and $Q_2(x, y) = x^2 + 3xy + 2y^2$ on $\mathbb{R}^2$. Choose the correct statements from below:

71. Let $\{u_n\}_{n=1}^{\infty}$ be a sequence of real numbers satisfying the following conditions:

72. Let $S$ be a subset of $\mathbb{R}$. Define $S^c$ as $\mathbb{R} \setminus S$. Consider the following statements:

1. 1 + 1 = 2
2. $\sqrt{2}$ is a rational number.
3. $\pi$ is an irrational number.
4. $\log_2(8) = 3$
73. Let \( S \) be an infinite set. Which of the following statements are true?
1. If there is an injection from \( S \) to \( \mathbb{N} \), then \( S \) is countable.
2. If there is a surjection from \( S \) to \( \mathbb{N} \), then \( S \) is countable.
3. If there is an injection from \( \mathbb{N} \) to \( S \), then \( S \) is countable.
4. If there is a surjection from \( \mathbb{N} \) to \( S \), then \( S \) is countable.

74. For \( n \geq 1 \), consider the sequence of functions
\[ f_n(x) = \frac{1}{2n+1}, \quad g_n(x) = \frac{x}{2n+1} \]
on the open interval \((0, 1)\). Consider the statements:
(i) The sequence \( (f_n) \) converges uniformly on \((0, 1)\).
(ii) The sequence \( (g_n) \) converges uniformly on \((0, 1)\).
Then,
1. (i) is true
2. (i) is false
3. (i) is false and (ii) is true
4. Both (i) and (ii) are true

75. Suppose \( f(x) \) is a sequence of continuous real-valued functions on \([0,1]\) satisfying the following:
(A) For all \( x \in \mathbb{R} \), \( f_n(x) \) is a decreasing sequence.
(B) The sequence \( f_n(x) \) converges uniformly to \( g(x) \). Let \( g(x) = \sum_{n=1}^{\infty} (-1)^{n+1} f_n(x) \) \( \forall x \in \mathbb{R} \).
Then,
1. \( f_n(x) \) is Cauchy with respect to the sup norm.
2. \( f_n(x) \) is uniformly convergent.
3. \( f_n(x) \) need not converge pointwise.
4. \( \exists M > 0 \) such that \( |g_n(x)| \leq M \), \( \forall x \in \mathbb{R} \).
76. Let \( f : [0, 2] \to \mathbb{R} \) be a continuous function and \( g(x) = f(x) + f(1/x), x \in [1, 2]. \)

1. Let \( f \) be a fixed function \( \mathbb{R} \to \mathbb{R} \) and \( g(x) = f(x) + f(1/x), x \in [1, 2]. \)
2. Let \( f \) be a fixed function \( \mathbb{R} \to \mathbb{R} \) and \( g(x) = f(x) + f(1/x), x \in [1, 2]. \)

77. Let \( f \) be a real valued continuously differentiable function of \((0, 1)\). Set \( g = f'' + f, \) where \( f' = -i \) and \( f'' \) is the derivative of \( f. \)

1. If \( g(a) > 0 \), then \( g \) crosses the real line from upper half plane to lower half plane at \( a \).
2. If \( g(a) > 0 \), then \( g \) crosses the real line from lower half plane to upper half plane at \( a \).
3. If \( g(a)g(b) < 0 \), then \( g(a), g(b) \) have the same sign.
4. If \( g(a)g(b) < 0 \), then \( g(a), g(b) \) have opposite signs.

78. A function \( F: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p \) is defined by \( F(x, y) = (Ax, y) \) where \( A \) is a matrix of size \( p \times (n+m) \). Let \( D_F(x, y) \) denote the derivative of \( F \) at \((x, y)\) which is a linear transformation from \( \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p \).

1. If \( x = 0 \), then \( D_F(x, 0) = 0 \).
2. If \( y = 0 \), then \( D_F(0, y) = 0 \).
3. If \( (x, y) \neq (0, 0) \), then \( D_F(x, y) \neq 0 \).
4. If \( x = 0 \) or \( y = 0 \), then \( D_F(x, y) = 0 \).
81. Suppose that \( R \) is a ring. Let \( \frac{1}{x^2 + 1} \) be a polynomial in \( \mathbb{Q}[x] \). Then which of the following is true?

1. \( R \) is a field.
2. \( R \) is an integral domain.
3. \( R \) is a local ring.
4. \( R \) is a principal ideal domain.

82. Let \( f(x) = x^2 - 105x + 12 \). Then which of the following are correct?

1. \( f(x) \) has exactly two real roots.
2. \( f(x) \) is irreducible over \( \mathbb{Q} \).
3. \( f(x) \) is reducible over \( \mathbb{Q} \).
4. \( f(x) \) is irreducible over \( \mathbb{R} \).

83. Let \( G \) be a group with the following property: Given any positive integers \( m, n \) and \( r \), there exist elements \( g \) and \( h \) in \( G \) such that \( \text{ord}(g) = m \), \( \text{ord}(h) = n \), and \( \text{ord}(gh) = r \). Then which of the following are necessarily true?

1. \( G \) is a finite group.
2. \( G \) is an infinite group.
3. \( G \) is abelian.
4. \( G \) is a non-abelian group.

84. If \( \sigma \) is a homomorphism of \( G \), then \( \sigma(\sigma(g)) = \sigma(g) \sigma(g) \). Then which of the following are correct?

1. \( \sigma \) is a bijection.
2. \( \sigma \) is a permutation.
3. \( \sigma \) is an isomorphism.
4. \( \sigma \) is a surjective homomorphism.
83. Let \( \alpha = \sqrt{2} \in \mathbb{R} \) and \( \xi = e^{\frac{2\pi i}{3}} \). Let \( \mathbb{K} = \mathbb{Q}(\alpha, \xi) \). Pick the correct statements from below:
1. There exists a field automorphism \( \sigma \) of \( \mathbb{K} \) such that \( \sigma(\mathbb{K}) = \mathbb{K} \) and \( \sigma \neq id \).
2. There exists a field automorphism \( \sigma \) of \( \mathbb{K} \) such that \( \sigma(\mathbb{K}) = \mathbb{K} \).
3. There exists a finite extension \( \mathbb{E} \) of \( \mathbb{Q} \) such that \( \mathbb{K} \subseteq \mathbb{E} \) and \( \sigma(\mathbb{K}) = \mathbb{E} \) for every field automorphism \( \sigma \) of \( \mathbb{E} \).
4. For all field automorphisms \( \sigma \) of \( \mathbb{K} \), \( \sigma(\mathbb{K}) = \mathbb{K} \).

84. The set \( X = \{(x_i, x_j) : x_i \in (0,1) \} \) is at least \( i \geq 1 \) of the following statements true?
1. \( X \) is connected
2. \( X \) is compact
3. \( X \) is one-to-one
4. \( X \) is open

85. Let \( A \) be a subset of \( \mathbb{R} \) satisfying \( A = \bigcap_{n \geq 1} V_n \), where for each \( n \geq 1 \), \( V_n \) is an open dense subset of \( \mathbb{R} \). Which of the following are correct?
1. \( A \) is a non-empty set
2. \( A \) is countable
3. \( A \) is uncountable
4. \( A \) is dense in \( \mathbb{R} \).

86. \( H \) is the upper half plane, that is, \( H = \{x + iy : y > 0\} \).
For \( z \in H \), which of the following are true?
1. \( \frac{z}{i} \in H \)
2. \( \frac{1}{z} \in H \)
3. \( \frac{z}{i} \in H \)
4. \( \frac{1}{z} \in H \)

87. Let \( f \) be a function from \( \mathbb{C} \) to \( \mathbb{C} \) given by \( f(z) = \frac{1}{z} \). Which of the following statements are correct?
1. \( f \) is continuous
2. \( f \) is onto
3. \( f \) is one-to-one
4. \( f \) is open

88. Let \( A \) be a subset of \( \mathbb{R} \) satisfying \( A = \bigcap_{n \geq 1} V_n \), where for each \( n \geq 1 \), \( V_n \) is an open dense subset of \( \mathbb{R} \). Which of the following are correct?
1. \( A \) is a non-empty set
2. \( A \) is countable
3. \( A \) is uncountable
4. \( A \) is dense in \( \mathbb{R} \).

89. Let \( f : \mathbb{C} \to \mathbb{C} \) be an analytic function. Then which of the following statements are true?
1. If \( |f(x)| \leq 1 \) for all \( x \in \mathbb{C} \), then \( f \) has infinitely many zeros in \( \mathbb{C} \)
2. If \( f \) is onto, then the function \( f(\cos z) \) is onto
3. If \( f \) is onto, then the function \( f(e^z) \) is onto
4. If \( f \) is one-one, then the function \( f(z^2 + 2) \) is onto
88. Let \( f(z) = 1 + z + z^2 \) and \( g(z) = e^z, z \in \mathbb{C} \), find the limit of \( f(z) \) as \( z \to \infty \).

89. Consider the entire functions \( f(z) = 1 + z + z^2 \) and \( g(z) = e^z, z \in \mathbb{C} \). Which of the following statements are true?

a) \( \lim_{|z| \to \infty} f(z) = \infty \)

b) \( \lim_{|z| \to \infty} g(z) = 0 \)

c) \( f^{-1}(\{z \in \mathbb{C} : |z| \leq R\}) \) is bounded for every \( R > 0 \)

d) \( g^{-1}(\{z \in \mathbb{C} : |z| \leq R\}) \) is bounded for every \( R > 0 \)

90. Let \( a_1 < a_2 < \cdots < a_{10} \) be given distinct natural numbers such that \( 1 \leq a_i \leq 100 \) for all \( i = 1, 2, \ldots, 10 \). Then which of the following are correct?

a) \( \exists i, j : 1 \leq i < j \leq 10 \) such that \( a_i + a_j \) is divisible by 11.

b) \( \exists i, j : 1 \leq i < j \leq 10 \) such that \( a_i + a_j \) is an even integer.

c) \( \exists i, j : 1 \leq i < j \leq 10 \) such that \( a_i + a_j \) is an odd integer.

Unit-3

91. Let \( u(x, t) \) be a solution of the wave equation

\[ u_{tt} + u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0, \]

subject to the initial conditions

\[ u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad x \in \mathbb{R}. \]

Then find the solution for

a) \( f(x) = \sin(x) \)

b) \( g(x) = \cos(x) \)

92. Let \( f : [0, 1] \to [0, 1] \) be a continuous function such that

\[ f(x) = x, \quad x \in [0, 1]. \]

a) Show that \( f(x) = x \) is the only solution.

b) Show that \( f(x) = x \) has no other solutions.

4-CH
92. Let \( f : [0, 1] \rightarrow [0, 1] \) be twice continuously differentiable function with a unique fixed point \( f(x_0) = x_0 \). For a given \( x_0 \in (0, 1) \), consider the iteration \( x_{n+1} = f(x_n) \) for \( n \geq 0 \).

If \( L = \max_{x \in [0,1]} |f'(x)| \), then which of the following are true?

1. If \( L < 1 \), then \( x_n \) converges to \( x_0 \).
2. \( x_n \) converges to \( x_0 \), provided \( L \geq 1 \).
3. The error \( e_n = x_n - x_0 \) satisfies \( |e_{n+1}| < L |e_n| \) for some \( C > 0 \).
4. If \( f'(x_0) = 0 \), then \( |e_{n+1}| < C |e_n|^2 \) for some \( C > 0 \).

93. Assume that \( u(x) \) satisfies the following boundary value problem

\[
(BVP) \quad \begin{cases}
  u'' + u' = 0 & x \in (0,1) \\
  u(0) = 0 \\
  u(1) = 1.
\end{cases}
\]

Let \( u(x) \) satisfy the boundary value problem

\[
(BVP) \quad \begin{cases}
  u'' + u' = 0, \\  u(0) = 0 \\
  u(1) = 1.
\end{cases}
\]

Consider the finite difference approximation to \( BVP \)

\[
(BVP)_h \quad \begin{cases}
  u_{j+1} - 2u_j + u_{j-1} = 0, & j = 1, \ldots, N-1 \\
  u_0 = 0 \\
  u_N = 1.
\end{cases}
\]

Here \( u_j \) is an approximation to \( u(x_j) \) where \( x_j = jh, j = 0, \ldots, N \) is a partition of \( [0,1] \) with \( h = 1/N \) for some positive integer \( N \). Then which of the following are true?

1. There exists a solution to \( (BVP)_h \) of the form \( u_j = ar^j + b \) for some \( a, b \in \mathbb{R} \) with \( r = 1 \) and \( r \) satisfying \( (2 + h)r^2 - 4r + (2 - h) = 0 \).
2. \( u_j = (r^j - 1)/(r^N - 1) \) where \( r \) satisfies \( (2 + h)r^2 - 4r + (2 - h) = 0 \) and \( r \neq 1 \).
3. \( u_j \) is monotonic in \( x \).
4. \( u_j \) is monotonic in \( j \).

94. \( y(0) = 0, y(1) = 0 \) \( \Rightarrow \) 침izi yon 255

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]

\( x \) \( \Rightarrow \) 침izi yon 256

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]

\( x \) \( \Rightarrow \) 침izi yon 256

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]

\( x \) \( \Rightarrow \) 침izi yon 256

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]

\( x \) \( \Rightarrow \) 침izi yon 256

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]

\( x \) \( \Rightarrow \) 침izi yon 256

\[
\begin{align*}
  \int y' \left( y'' - (y')^2 \right) dx &= \int 2 \left( y'' - (y')^2 \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - \int 2 \left( y'' \right) \left( y' \right) dx \\
  &= 2 \left( y'' \right) \left( y' \right) - 2 \left( y'' \right) \left( y' \right) + C.
\end{align*}
\]
94. Consider the functional
\[ I[y] = \int_{a}^{b} \left[ (y')^2 - (y')^3 \right] dx \]
subject to
\[ y(a) = y(b) = 0. \]
A broken extremal is a continuous extremal whose derivative has jump discontinuities at a finite number of points. Then which of the following statements are true?
1. There are no broken extremals and \( y = 0 \) is an extremal.
2. There is a unique broken extremal.
3. There exist more than one and finitely many broken extremals.
4. There exist infinitely many broken extremals.

95. The broken extremals of the functional
\[ I[y] = \int_{a}^{b} \left[ (y')^2 - (y')^3 \right] dx \]
subject to \( y(a) = y(b) = 0 \) and \( y'(a) = y'(b) = 0 \).
1. \( x^2 + 2x^3 - 3x^2 \)
2. \( x^2 + 2x^3 - 3x^2 \)
3. \( x^2 + 2x^3 - 3x^2 \)
4. \( x^2 + 2x^3 - 3x^2 \)

96. \( \phi(x) = 1 - 2x - 4x^2 + \int_{0}^{x} \left[ 3 + 6(x - t) - 4x - (t)^3 \right] \phi(t)dt \)
\( \phi(0) \) is finite, \( \phi(0) = \phi(0) \)
1. 1
2. 2
3. 3
4. 4

97. A characteristic number and the corresponding eigenfunction of the homogeneous Fredholm integral equation
\[ K(x,t) = \int_{0}^{1} \left( x(t) - 1 \right) \left( t(x - 1) - 1 \right) dt \]
1. \( \lambda = -\beta, \phi(x) = \sin x \)
2. \( \lambda = -2\pi^2, \phi(x) = \sin 2\pi x \)
3. \( \lambda = -5\pi^2, \phi(x) = \sin 5\pi x \)
4. \( \lambda = -4\pi^2, \phi(x) = \sin 4\pi x \)

98. The solution of
\[ \phi(x) = 1 - 2x - 4x^2 + \int_{0}^{x} \left[ 3 + 6(x - t) - 4x - (t)^3 \right] \phi(t)dt \]
then \( \phi(0) \) is equal to
1. 1
2. 2
3. 3
4. 4

99. \( \phi(x) = \begin{cases} \pi - x & 0 \leq x \leq 1 \\ x & 1 < x \leq 2 \end{cases} \)
\( \phi(0) = \pi - 0 = \pi \)
\( \phi(1) = 1 \)
1. \( \phi(x) = \pi - x \)
2. \( \phi(x) = \pi - x \)
3. \( \phi(x) = \pi - x \)
4. \( \phi(x) = \pi - x \)
98. Consider a point mass of mass $m$, which is attached to a mass-less rigid rod of length $a$. The other end of the rod is made to move vertically such that its downward displacement from the origin at time $t$ is given by $z(t) = z_0 \cos \omega t$. The mass is moving in a fixed plane and its position vector at time $t$ is given by $r(t) = (a \sin \theta(t), z_0 + a \cos \theta(t))$. Then the equation of motion of the point mass is

$$\begin{align*}
1. & \quad \frac{d^2 \theta}{dt^2} + (g - z_0 \omega^2 \cos(\omega t))\sin \theta = 0 \\
2. & \quad \frac{d^2 z}{dt^2} + (g - z_0 \omega^2 \cos(\omega t)) \cos \theta = 0 \\
3. & \quad \frac{d\theta}{dt} + (g + z_0 \omega^2 \sin(\omega t)) \sin \theta = 0
\end{align*}$$

99. Find the differential equation of the motion in the general case.

99. The method of variation of parameters to solve the differential equation $y'' + p(x)y' + q(x)y = r(x)$, where $r(x)$ is not zero, is a method of solving non-homogeneous linear differential equations.

100. The method of variation of parameters to solve the differential equation $y'' + p(x)y' + q(x)y = r(x)$, where $r(x)$ is not zero, is a method of solving non-homogeneous linear differential equations. Which of the following statements are necessarily true?

1. The Wronskian of $y_1$ and $y_2$ is never zero in $I$.
2. $y_1$, $y_2$, and $y_1y_2$ are twice differentiable.
3. $y_1$ and $y_2$ may not be twice differentiable, but $y_1^2 + y_2^2$ is twice differentiable.
4. The solution set of $y'' + p(x)y' + q(x)y = r(x)$ consists of functions of the form $ay_1 + by_2 + y_0$, where $a, b \in \mathbb{R}$ are arbitrary constants.
101. Consider the eigenvalue problem
\[ y'' + ay = 0 \quad \text{for} \quad x \in (-1,1) \]
\[ y(-1) = y(1) \]
\[ y'(-1) = y'(1). \]
Which of the following statements are true?
1. All eigenvalues are strictly positive.
2. All eigenvalues are non-negative.
3. Distinct eigenfunctions are orthogonal in \([-1,1].
4. The sequence of eigenvalues is bounded above.

102. Let \( u(x,t) = x^2 + t^2 \) be a solution of the heat equation
\[ u_t - u_{xx} = 0 \]
Then the solution is singular at \( (0,0) \).
2. The given space curve \( (x,t,u) = (t^2, t^2, t^2) \) is not a characteristic curve at \( (0,0) \).
3. There is no base characteristic curve in the \((x,t)\) plane passing through \((0,0)\).
4. A necessary condition for the IVP to have a unique \( C^1 \) solution at \((0,0)\) does not hold.

Unit-4

103. Let \( X_1, X_2, ..., X_n \) be independent random variables following a common continuous distribution \( F \), which is symmetric about 0. For \( t = 1, 2, ..., n \), define
\[ S_t = \begin{cases} 1 & \text{if } X_t > 0 \\ -1 & \text{if } X_t < 0 \text{ and} \\ 0 & \text{if } X_t = 0 \end{cases} \]
\[ R_t = \text{rank of } |X_t| \in \{ |X_1|, ..., |X_n| \} \] Which of the following statements are correct?
1. \( S_1, S_2, ..., S_n \) are independent and identically distributed.
2. \( R_1, R_2, ..., R_n \) are independent and identically distributed.
3. \( S = (S_1, ..., S_n) \) and \( R = (R_1, ..., R_n) \) are independent.
4. The distribution of \( T = \sum_{t=1}^{n} S_t R_t \) does not depend on the functional form of \( F \).
104. Suppose $Y|\theta \sim \text{Poisson}(\theta)$, $\theta > 0$ and prior density $\tau$ of $\theta$ is given by $\tau(\theta) \propto \theta^\alpha e^{-\beta \theta}$, where $\alpha > 0$ and $\beta > 0$ are hyperparameters. Which of the following are true?

1. Marginal distribution of $Y$ is hypergeometric.
2. Posterior distribution of $\theta$ given $Y = y$ is Gamma.
3. $\tau$ is a conjugate prior.
4. Bayes' estimate of $\theta$ for squared error loss function is $\frac{y}{\alpha} + \frac{1}{\beta}$.

105. Consider a linear model $Y_{i} = X_{i}^{\top} \beta + \epsilon_{i}$, where $\epsilon_{i} \sim N(0, \sigma^{2})$. One wants to choose the design matrix $X$ such that its elements take values in the set $\{-1, 0, 1\}$. Now, consider the following four choices of $X$:

$X_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$X_2 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

$X_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

Which of the following statements are true?

1. For all three choices of $X$, $\beta = (\beta_0, \beta_1, \beta_2) \sim N(0, \sigma^2)$ is estimable.
2. For all three choices of $X$, $\beta_0$ and $\beta_1$ are uncorrelated.
3. $X_1$ is a better choice than $X_2$.
4. $X_2$ is a better choice than $X_3$.

106. $X_1, X_2, \ldots, X_{10}$ are i.i.d. $N(0, 1)$ random variables. Let the following statements be true:

1. $P(X_1 > X_2 + \ldots + X_{10}) = \frac{1}{2}$
2. $P(X_1 > X_2 + X_3) = \frac{1}{2}$
3. $P(\sin(X_1) > \sin(X_2) + \sin(X_3)) = \frac{1}{2}$
4. $P(\cos(X_1) > \cos(X_2) + \cos(X_3) + \ldots + \cos(X_{10})) = \frac{1}{2}$
106. Suppose that \( X_1, X_2, \ldots, X_{10} \) are \( N(0, 1) \) random variables. Which of the following statements are correct?

1. \( P(X_1 > X_2 + X_3 + \cdots + X_{10}) = \frac{1}{2} \)
2. \( P(X_1 > X_2 + X_3 + \cdots + X_{10}) = \frac{1}{2} \)
3. \( P(\sin(X_1) > \sin(X_2) + \sin(X_3) + \cdots + \sin(X_{10})) = \frac{1}{2} \)
4. \( P(\sin(X_1) > \sin(X_2 + X_3 + \cdots + X_{10})) = \frac{1}{2} \)

107. एक समान बंदरों \( U(0, 2) \) तथा \( U(1, 5) \) के लालगुण श्रेणी पर विचार कीजिए।

माना कि \( P(0 < \theta < 1) आंशिक दूरी की पूर्व प्रायिकता है जिसका \( U(0, 2) \) बंदर है। विद्युत-\( \frac{1}{2} \) प्रायिकता पर विचार करें कि दूरी में से किस बंदर से कब्ज़ा होगा?

1. \( \eta < \frac{1}{3} \) के लिए, \( \text{वैलिड} \) है।
2. \( \eta > \frac{1}{3} \) के लिए, \( \text{वैलिड} \) है।
3. \( \eta = \frac{1}{3} \) \( \text{वैलिड} \) है।
4. \( \eta \) का यह भी \( \text{है} \), \( \text{वैलिड} \) है।

108. Suppose \( n \geq 2 \) units are drawn from a population of \( N(\mu, \sigma^2) \) units sequentially as follows. A random sample \( U_1, U_2, \ldots, U_k \) of size \( N \) is drawn from \( U(0, 1) \). The \( k \)-th population unit is selected if \( U_k < \frac{\mu - \bar{U}}{S} \), where \( \bar{U} = \frac{U_1 + \cdots + U_k}{k} \) and \( S = \sqrt{\frac{1}{k-1} \sum_{i=1}^{k} (U_i - \bar{U})^2} \). Then,

1. The probability of inclusion of the \( k \)th unit in the sample is \( \frac{n}{n+k-1} \).
2. The probability of inclusion of the first and \( k \)th unit in the sample is \( \frac{n}{n+k-1} \).
3. The probability of not including the \( k \)th unit and including the first unit in the sample is \( \frac{n-1}{n+k-1} \).
4. The probability of including the first unit but not including the \( k \)th unit in the sample is \( \frac{n(n-1)}{(n+k-1)(n+k-2)} \).

109. लेखाकार तथा भाषा के लक्षण \( A, B, C \) तथा \( D \) के कारण ब्रिटिश विद्यार्थी उन्हें केवल A तथा B को "वैश्विक" के लिये, तथा C तथा D को "वैश्विक-2" के लिये, ब्रिटेन C को "वैश्विक-3" के लिये रखा गया है। तब प्रति वर्ष \( \text{वैश्विक} \) विद्यार्थी
1. A project is taken for implementation.

2. A project is taken for implementation.

3. A project is taken for implementation.

4. A project is taken for implementation.

109. Consider a block design with three blocks and four treatments A, B, C, and D where only A and B are allotted to block-1, only A and B are allotted to block-2, and only C is allotted to block-3. Then the resulting block design is

1. Incomplete and not connected
2. Incomplete and not connected balanced and connected
4. Neither balanced nor connected

110. Suppose \( X \) is a positive random variable with the following probability density function

\[
f(x) = (\alpha x^{\alpha-1} + \beta x^{\beta-1})e^{-x^\alpha - x^\beta}; x > 0, \]

for \( \alpha > 0 \) and \( \beta > 0 \). Then the hazard function of \( X \) for some choices of \( \alpha \) and \( \beta \) can be

1. An increasing function
2. A decreasing function
3. A constant function
4. A non-monotonic function

111. A parallel system has \( n \geq 1 \) identical components. The lifetimes of the \( n \) components are independent identically distributed exponential random variables with mean \( \lambda \). If the lifetime of the system is denoted by \( T \), then which of the following statements are true?

1. The mode of \( T \) is 0 for some \( n \).
2. The mean of \( T \) is less than or equal to \( \lambda \) for all \( n \).
3. The mean of \( T \) is greater than or equal to \( \lambda \) for all \( n \).
4. The median of \( T \) is greater than or equal to \( \lambda \) for some \( n \).

112. Suppose \( ABC \) is a triangle on the \( xy \)-plane with centroid \( D \). Which of the following points can NEVER be a minimizer of the function \( 7x - 10y + 1 \) as \((x,y)\) runs over the triangle \( ABC \)?

A. \( A \)  B. \( B \)  C. \( C \)  D. \( D \)

113. Suppose \( x_1, x_2, \ldots, x_n \) is a iid random variable. \( x_0 \) \( (0,2) \) on \( \text{eg} \) is a minimizer of the function \( 7x - 10y + 1 \) as \((x,y)\) runs over the triangle \( ABC \)?

A. \( A \)  B. \( B \)  C. \( C \)  D. \( D \)
3. $M_n \to 2$ almost surely

4. $\frac{S_n}{n} \to X_0$ in distribution

113. Suppose $X_0, X_1, \ldots, X_n$ is a random sample from the uniform distribution on $(0, 2)$ and $M_n = \max \{X_1, X_2, \ldots, X_n\}$ for each positive integer $n$. Then which of the following statements are true?
1. $M_n \to 2$ almost surely
2. $M_n \to 2$ in probability
3. $M_n \to 2$ in distribution
4. $\frac{S_n}{n} \to M_n$ converges in distribution to normal distribution

114. Discuss whether $X_1, X_2, \ldots$ is a Markov chain with state space $S$. For any $i, j \in S$, let $p^n_{ij}$ denote the $n$-step transition probability of going from $i$ to $j$. Let $d(i)$ denote the period of state $i$ ($i \in S$). Which of the following statements are correct?
1. If $d(i) = d(j)$ then $\lim_{n \to \infty} p^n_{ij} = 0$
2. If $d(i) = d(j)$ then $p^n_{ij} = 0$ and $\lim_{n \to \infty} p^n_{ij} = 0$ for some $n, m \geq 2$
3. If $p^n_{ij} = 0$ and $p^m_{ij} = 0$ for some $n, m \geq 2$, then $d(i) = d(j)$
4. $\lim_{n \to \infty} p^n_{ij} > 0$ implies $d(i) = d(j)$

115. Let $X_0$ be a Markov chain with state space $S$. For any $i, j \in S$, let $p^n_{ij}$ denote the $n$-step transition probability of going from $i$ to $j$. Let $d(i)$ denote the period of state $i$ ($i \in S$). Which of the following statements are correct?
1. If $d(i) = d(j)$ then $\lim_{n \to \infty} p^n_{ij} = 0$
2. If $d(i) = d(j)$ then $p^n_{ij} = 0$ and $\lim_{n \to \infty} p^n_{ij} = 0$ for some $n, m \geq 2$
3. If $p^n_{ij} = 0$ and $p^m_{ij} = 0$ for some $n, m \geq 2$, then $d(i) = d(j)$
4. $\lim_{n \to \infty} p^n_{ij} > 0$ implies $d(i) = d(j)$

116. Consider a Markov chain with transition probability matrix $P$. Identify the correct statements.

$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$
117. Let \( (X_1, X_2) \) be a random sample from the distribution with p.d.f.
\[
f_0(x) = \begin{cases} 
\frac{1}{2} x^2, & 0 < x < 1, \\
0, & \text{otherwise}
\end{cases}
\]
where \( \theta > 0 \). Then which of the following statements are correct?
1. \( \prod X_i \) is sufficient for \( \theta \).
2. \( -\frac{1}{\theta} \sum X_i \ln X_i \) is sufficient for \( \theta \).
3. \( \prod X_i \) is a maximum likelihood estimate for \( \theta \).
4. \( -\frac{1}{\theta} \sum X_i \ln X_i \) is a maximum likelihood estimate for \( \theta \).

119. \((-2, -1, 1, 2)\) are the possible probability functions for the random variable \( X \) with probability mass function \( p_0(x) \). Determine the values of \( \theta \) and \( \beta \).

\[\begin{array}{c|cccc}
\hline
x & -2 & -1 & 1 & 2 \\
\hline
\theta = \theta_0 & 0.05 & 0.4 & 0.3 & 0.2 \\
\theta = \theta_1 & 0.2 & 0.4 & 0.5 & 0.2 \\
\hline
\end{array}\]

118. \( X_1, X_2, \ldots, X_n \) be a random sample from the distribution with p.d.f. \( f_0(x) \). Let \( \prod X_i \) be sufficient for \( \theta \).

\[\begin{align*}
\text{Let } X_1, X_2, \ldots, X_n \text{ be a random sample from the distribution with p.d.f. } f_0(x), \text{ where } \theta > 0. \text{ Then which of the following are true?} \\
1. \prod X_i \text{ is sufficient for } \theta. \\
2. -\frac{1}{\theta} \sum X_i \ln X_i \text{ is sufficient for } \theta. \\
3. \prod X_i \text{ is a maximum likelihood estimate for } \theta. \\
4. -\frac{1}{\theta} \sum X_i \ln X_i \text{ is a maximum likelihood estimate for } \theta.
\end{align*}\]
The aim is to test $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$. Which of the following statements are correct?

1. The test procedure with critical region \( \{ x = 2 \} \) is a most powerful test of size 0.05.
2. The test procedure with critical region \( \{ x = -2 \} \) is a most powerful test of size 0.05.
3. The test procedure with critical region \( \{ x = 1 \} \) is not a most powerful test of its size.
4. The test procedure with critical region \( \{ x = 1 \} \) is not a most powerful test of its size.

120. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from uniform distribution on $(0, \theta + 1)$, where $\theta \in \mathbb{R}$ is an unknown parameter. Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the corresponding order-statistics. Which of the following are $100(1 - \alpha)\%$ confidence intervals for $\theta$?

1. $(\infty, X_{(n)} - a^{1/n})$
2. $(X_{(1)} + a^{1/n} - 1, \infty)$
3. $(X_0 + \frac{a}{n} - 1, X_0 - \frac{a}{n})$
4. $(\infty, X_1 - a)$
FOR ROUGH WORK